Price Destabilizing Speculation: The Role of Strategic Limit Orders*

Suman Banerjee†
Ravi Jagannathan‡
Kai Wang§

January 31, 2024

Using a two-period model of a commodity market with many atomistic consumers and two strategic sellers, we show that a speculator with access to storage can lower the market-clearing price while buying and raise the market-clearing price while selling by clever use of limit, stop-loss, and market orders, and profit from it. This creates price volatility even though there is no demand or supply uncertainty, and all market participants act rationally. Such speculative activity makes the strategic sellers worse off and consumers better off. As the number of strategic sellers becomes large, consumers also can be worse off.

Keywords: Endogenous Price Volatility, Limit Orders, Price Destabilization, Speculator, Strategic Storage, Stop-Loss Order.

JEL Codes: G32, G34, G38, K20.

I Introduction

The classical academic view, based on the assumption that all agents behave competitively, is that stable prices are socially beneficial and profitable speculation involves buying low and selling high. Buying when the price is low would lift the price up and selling when the price is high would push the price down, resulting in a stabilizing effect. A speculator who buys and sells without regard to the price will introduce volatility due to the price impact of trades thereby destabilizing prices, but such noise trading will result in losses. This classical view is

*We are grateful for helpful comments from Sugata Bhattacharya, Sudipto Dasgupta, Benjamin Herbert, Anand Goel, Gustavo Grullon, David Hirshleifer, Bob Kimmel, Albert Kyle, Ron Masulis, Vikram Nanda, Tom Noe, Marco Pagano, David Reeb, Alexander Rodivilov, Ailsa Röell, Duane Seppi, Vikrant Vig, Vish Viswanathan, Yajun Wang, Jingshu Wen, participants at the American Finance Association Virtual Meeting 2020, and seminar participants at Australian National University, Florida International University, Indian Institutes of Management at Calcutta, Bangalore, & Udaipur, Indian Statistical Institute, Krea University, La Trobe University, Northwestern University, Stevens Institute of Technology, University of Adelaide, NBER Economics of Commodity Markets, University of Central Florida, University of New South Wales, University of Wyoming, and West Virginia University. The usual disclaimer applies.

†School of Business, Stevens Institute of Technology, Email: sbanerj2@stevens.edu.
‡Kellogg School of Management, Northwestern University, and NBER Email: rjaganna@kellogg.northwestern.edu.
§Kellogg School of Management, Northwestern University, Email: kai.wang2@kellogg.northwestern.edu.

summarized by the following quote from Friedman (1953): "*People who argue that speculation is generally destabilizing seldom realize that this is largely equivalent to saying that speculators lose money since speculation can be destabilizing in general only if speculators on the average sell when the currency is low in price and buy when it is high.*" There are a number of examples of destabilizing speculation in the literature, clarifying the limitations of this classical view.[1] Notable among them is Hart and Kreps (1986b), who provide an example where speculation is destabilizing but profitable on average even though all agents have the same priors and access to information and behave competitively.

However, the market for many of the commodities has a few dominant producers, with behavior closer to an oligopoly than perfect competition. The market for crude oil, coffee, and rare earth would be examples. In such markets, as Newbery (1984b) argues, dominant producers will have an incentive for stable prices, which is consistent with producers often blaming speculators as the reason for high and volatile commodity prices.[2] A natural question is whether profitable speculation can destabilize prices in oligopolistic markets when all agents are rational a question that continues to be of interest to policymakers.[3] We address this question in this paper.

Regulators are likely to act when they notice destabilizing speculation, and therefore any such speculative behavior is likely to occur sporadically relying on different mechanisms at differnt points in time, and lasting only for short periods of time. While the literature identifies scenarios where speculation can be destabilizing, necessarily they can not be comprehensive and complete. In the literature destabilizing speculation often occurs in economies with asymmetric information, agency problems, large demand/supply shocks, or potentially large policy changes. [4] Further, the literature suggests speculators are likely to have more influence in illiquid mar-

[1]See Newbery (1987) for a discussion of when futures markets destabilize spot prices.

[2]CNN Monday, 7 Dec 2011, reported OPEC Secretary-General Abdulla Salem El Badri saying "speculators are at least partly to blame for high oil prices – not any lack of supply on world markets."

[3]The Staff Report of the United States Senate, Senate (2014), mentions "Past investigations have presented case studies on pricing gasoline; exposing a $6 billion manipulation of natural gas prices by a hedge fund called Amaranth; closing the Enron loophole impeding energy market oversight; tracing excessive speculation in the crude oil and wheat markets; exposing the increased role of mutual funds, exchange-traded funds, and other financial firms in commodity speculation; and revitalizing position limits as tools to combat market manipulation and excessive speculation." In her 2022 Senate address, Senator Maria Cantwell hints toward intentional destabilization of transportation fuel prices, as indicated in Senator Cantwell (2022), and emphasizes the significance of transparency within the transportation fuel market.

[4]See Hart and Kreps (1986a), Goldstein and Guembel (2008), Xuanming (2010), Peng and Röell (2014), Lof (2015), Włodarczyk and Szturo (2019), Büyükşahin and Harris (2016), Kliesen (2021), Lof (2015), Włodarczyk and Szturo (2019), Büyükşahin and Harris (2016), and Kliesen (2021), Dai, Xiao, Su, and Ayub (2022), Chen and Wang (2022), and Xiong, Duc Huynh, and Wang (2022), Allen and Gale (1992), Opler and Titman (1994), Scheinkman and Xiong (2003), Eichengreen, Rose, and Wyplosz (1995), Attari, Banerjee, and Noe (2006), Hetzel (2012), Gan, Tsoukalas, and Netessine (2021), and Buraschi and Whelan (2022).

kets,[5] and instability in one market can spill over into other markets. [6] We add to this literatrue by showing the important role that limit orders and stop loss orders can play in destabilizing speculation even in the absence of demand/supply shocks, informational asymmetry, agency frictions, or economic events that act as catalysts.

We model a two-period commodities market with two strategic sellers, price-taking atomistic consumers, symmetric information, no uncertainties, and where all agents act rationally. The commodity can be stored but consumers are atomistic with little storage capacity. In the benchmark case without any speculators, prices are the same across periods. We show that when a large speculator with access to storage enters the market, prices become volatile. The speculator has no use-value for the commodity and buys the commodity in period 1 to build up his inventory which he sells in period 2. Each agent knows how other agents will behave. For example, each strategic seller knows that the speculator is buying in period 1 to sell the commodity in period 2 and both the strategic sellers are fully aware of the objective function and strategy set of the speculator. Hence, the strategic sellers can figure out in advance the consequences of the speculator's participation on their profits.

We show that the speculator is able to *lower* the market-clearing price when buying to build an inventory in period 1, and able to *raise* the market-clearing price when selling the acquired inventory in period 2. The speculator does this by changing the aggregate demand curve that the two strategic sellers take as given through clever use of market, limit, and stop-loss orders. This generates price volatility that the speculator profits from it. The way the speculator chooses the limit buy price and quantity ensures that if one strategic seller participates then the participating strategic seller is better off. But if both strategic sellers participate, both are worse off in terms of aggregate profits that they earn, but still they both choose to participate. Strategic sellers voluntarily supply the inventory to the speculator in period 1, compete with the speculator in period 2, and as a result, earn lower profits, even though they know the speculator's strategy. Both sellers will complain about the adverse impact on prices due to speculation.

When the speculator has the ability, in period 2, to freely dispose of parts of the inventory that he acquired in period 1, the speculator trades using a combination of market and stop-loss orders. The resulting market prices are more volatile. The speculator earns a higher profit.

[5]See Kim (2015), Bianco (1997), Hertzberg (2018), and Bohl and Sulewski (2019).
[6]See(Liu, Z., and Zhao, 2015, see, e.g.,). Hertzberg (2018).

Overall, consumers are better off when the two periods are taken together but they will naturally complain about high prices due to speculation in period 2. The speculator's profit is lower than the combined loss of the two strategic sellers, which would inhibit the desire of strategic sellers to engage in speculation.

Our main results remain unchanged when there are more than two strategic sellers. However, as the number of strategic sellers becomes large but not too large, consumers can also be worse off.[7] This is due to the speculator's inventory at the beginning of period 2 being large relative to each strategic seller's supply, which gives the speculator a greater implicit "bargaining" power in period 2. The resulting period 2 market-clearing price is higher relative to the two strategic seller cases. With many strategic sellers, the benchmark market-clearing price without the speculator is already low. That limits the speculator's ability to lower the market-clearing price in period 1 while buying. Therefore, when there are many strategic sellers, the decline in consumers' welfare due to the high period 2 market-clearing price outweighs the benefit to consumers' welfare due to the drop in period 1 market-clearing price.

Our results continue to hold when we allow for two speculators who compete with each other provided the aggregate storage capacity available to the two speculators taken together is limited. High storage costs will deter destabilizing speculation by eroding profits.

One notable implication arising from our findings is that the likelihood of experiencing destabilizing speculation in the commodity market is heightened in situations where the costs of borrowing are low and readily available storage facilities abound. In such circumstances, market participants are incentivized to engage in speculative activities that can potentially destabilize prices. The combination of low borrowing costs and accessible storage facilities creates an environment conducive to aggressive trading strategies, as participants can easily leverage their positions and store commodities for extended periods.

The ease with which participants can access funds at a low cost encourages them to adopt large strategic positions, contributing to increased market volatility. Additionally, the ready availability of storage facilities allows for the accumulation of large inventories, providing speculators with the means to exert greater influence on future market dynamics. Therefore, our research underscores that the interplay between low borrowing costs and abundant storage resources serves as a catalyst for the manifestation of destabilizing speculation within the com-

[7] As the number of strategic sellers becomes very large, profitable speculation is not feasible in our model economy, and consumes will be better off relative to the case with two strategic sellers.

modity market. This insight has implications for policymakers, market regulators, and industry participants, emphasizing the importance of monitoring and managing these factors to mitigate the risks associated with excessive speculation and potential market disruptions.

In contrast to high storage costs, which act as a deterrent to speculative activities, our analysis reveals that uncertainty surrounding consumers' demand in period 2 does not alter our conclusions, particularly in scenarios where producers and the speculator exhibit risk-neutrality. This distinction underscores the robustness of our findings in the face of demand uncertainty and provides valuable insights into the dynamics of speculative behavior in the context of our model.

Our research makes a significant contribution to the extensive body of literature that scrutinizes the ramifications of speculation on price stability. This line of inquiry can be traced back to the seminal work of Adam Smith (1976), who underscored the pivotal role of expectations in the realm of speculation, complemented by a thorough historical analysis of commodity speculation provided by Cowing (1965).

In a landmark contribution, Muth (1961) advocates for rational expectations, presenting an illustrative example that delves into the economic consequences of commodity speculation in the presence of storage. Expanding on this foundation, Hart and Kreps (1986b) posit that speculation, when accompanied by access to storage and predicated on noisy information about future demand, can be simultaneously profitable and destabilizing. Their model establishes a link between prices at different points in time through storage, augmenting current price volatility while leaving future price volatility unaffected due to shocks rendering future demand inelastic. Additionally, Stein (1987) assert that the entry of speculators, when market participants learn from prices, can complicate the learning process and contribute to price destabilization.

Furthermore, Newbery (1984a) investigates the impact of market power on speculative storage and its repercussions on price stability. In scenarios characterized by production uncertainty and a linear demand response to price changes, their findings indicate that significant suppliers strategically prefer more stable prices, thereby enhancing overall price stability.

Our study extends this literature by unveiling the potential destabilizing effects introduced by a substantial speculator in such markets. This stands in contrast to the findings of Kawai (1983), whose mean-variance rational expectations framework demonstrates that speculator participation can amplify futures price volatility in the presence of production and storage uncertainties. Remarkably, in the absence of supply shocks or storage, futures trading tends to stabilize prices, aligning with the observations of Turnovsky (1983). Moreover, Chari, Jagannathan, and Jones (1990) emphasizes that even in scenarios devoid of production uncertainties and storage, speculator participation in futures markets may elevate spot price variance. In our model, where there are no demand and supply shocks, speculation itself creates endogenous price volatility.

Our research also contributes to the literature exploring profitable price destabilization, as investigated by Hart (1977). Examining scenarios where some market participants possess sophisticated knowledge, Chatterjea and Jarrow (1998) highlight the potential for price destabilization in the Treasury auction market when "when-issued" market dealers utilize their information on net order flow. Analogous strategies leading to commodity price destabilization are discussed by Cooper and Donaldson (1998).

Within our framework, the use of limit and stop-loss orders emerges as pivotal in facilitating profitable speculation, with the noteworthy insight that stop-loss orders, by augmenting supply at lower prices, have a destabilizing impact.

Our work aligns with the strategic storage literature initiated by (see, e.g., De Long, Shleifer, Summers, and Waldmann, 1990; Deaton and Laroque, 1996), who empirically explores the influence of competitive storage on price dynamics. Expanding on this, Basak and Pavlova (2016) demonstrate that the financialization of commodities can contribute to heightened price volatility, as encapsulated in Cheng and Xiong (2014)'s comprehensive survey of the relevant literature.

While financialization can enhance risk sharing and information discovery in commodity markets by attracting speculators, as highlighted by Cheng and Xiong (2014), a caveat emerges. If speculators must unwind their long commodity positions due to abrupt price drops in other markets, their participation can transmit unrelated shocks to commodity markets, escalating price volatility and introducing a destabilizing effect (see, e.g., Boyd, Harris, and Li, 2018). Our study contributes by illustrating that the entry of a significant speculator can endogenously generate price volatility.

Subsequent sections of our paper delve into the intricacies of our benchmark model economy with duopolistic strategic sellers (Section II.A) and analyze the destabilizing impact of a large speculator (Section II.B). We extend the model to encompass scenarios with multiple strategic sellers (Section III.A), consider demand uncertainty (Section III.B), and explore situations involving more than one speculator (Section III.C). Our conclusions are presented in Section

IV.

II The Model Economy

The economy consists of two periods. There are two dates within each period, beginning and end. There is a single good in the economy, which we call "widgets" for convenience. There is a market for widgets in each period. Agreement to buy and sell widgets are entered into at the beginning of each period for delivery and settlement at the end of the period, when consumption takes place. We assume that commodity prices are denoted in hypothetical unit of accounts called "dollars", and the interest rate is zero. At the beginning of each period t, $t = 1, 2$, buyers and sellers enter into forward contracts in the market, for delivery at the end of the same period, t. Consumption takes place at the end of each period t, $t = 1, 2$. A buyer of a widget enters into a forward contract at the beginning of period t, $t = 1, 2$, to pay p_t dollars to the seller upon delivery of the widget at end of period t.There are three types of participants in the commodity market:

(a) A large number of infinitely small consumers distributed on the unity of interval, each with a different reservation price for widgets. A given consumer will buy one unit of the widget if the price is equal to or below her reservation price. This gives rise to the consumers' aggregate demand curve, which we assume is linear, as given below,

$$\text{(II.1)} \qquad \boldsymbol{p}_t(Q_t) = a - b\,Q_t \qquad t = \{1, 2\},$$

where Q_t is the aggregate demand from the price-taking consumers.

(b) Two identical strategic sellers who participate in the market for widgets in period 1 and period 2. Each seller decides how much to produce in each of the two periods, taking the aggregate demand schedule and the supply of the other strategic seller's supply as given, so as to maximize her aggregate profits over the two periods.

(c) A large speculator with access to storage. The speculator lacks the ability to produce widgets. He buys widgets in the market in period 1, stores, and sells them in the market in period 2.

We assume that the demand and supply curves for the widgets are known, and there are no uncertainties. The speculator's buy-sell decisions are common knowledge, and agreements to buy and sell are fulfilled with no defaults. In this environment, if a spot market is introduced

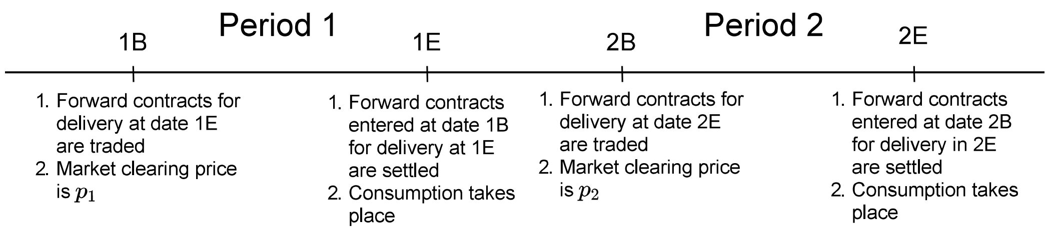

FIG. 1. Timeline of the two-period commodity market model

at the end of a period, the spot price would be the same as the forward price that prevailed at the beginning of the period. The temporal evolution of events is depicted in Figure 1.

In what follows we first characterize the equilibirum without the speculator which we use as the benchmark.

II.A Duopoly Equilibrium with no Speculators

We denote the two strategic sellers as A and B. In each period t, $t = 1, 2$, each strategic seller decides how much to produce and sell in the market. We denote the amount sold by the two strategic sellers, A and B, in period 1 as q_{A1}, q_{B1} respectively. Similarly, we denote the amount sold in period 2 as q_{A2}, q_{B2}. In each period t, $t = 1, 2$, strategic seller A chooses the supply q_{At} so as to maximize the profit in period t, taking q_{Bt}, and the demand schedule as given. Similarly, strategic seller B optimally chooses the quantity q_{Bt}, t, $t = 1, 2$. In equilibrium, the aggregate demand will be equal to the aggregate supply for each period of t, $t = 1, 2$. We use boldface letters to denote functions and normal letters to denote specific values taken by the variables. For expositional convenience, we assume that the marginal cost of production of all strategic sellers is zero.

In equilibrium, prices and quantities are the same in both periods 1 and 2. Further, the equilibrium is symmetric, and both strategic sellers will supply the same amount of goods and make the same profit. We can therefore drop the subscript i and t from the benchmark equilibrium supplies, demands, and prices. Denote the equilibrium supply of each strategic seller as q^*, the market-clearing price as p^*, and each strategic seller's profit π^* where,

$$q^* = \frac{a}{3\,b}, \quad p^* = \frac{a}{3}, \quad \text{and} \quad \pi^* = \frac{a^2}{9\,b}. \tag{II.2}$$

The equilibrium supply is determined by solving the reaction functions of all strategic sellers simultaneously. The equilibrium prices and supplies are the same in both periods.

Example 1:

Let the intercept, a, and slope, b, of the inverse demand function of the atomistic consumers, be 90 and 1 respectively. The two dashed lines in Figure 2 depict the best response functions of the strategic sellers when there is no speculator. The equilibrium is given by the point where the two lines intersect. Equation (II.2) implies that the equilibrium supply is $q^* = \frac{a}{3\,b} = 30$, the equilibrium price is $p^* = 30$, and the equilibrium profit of each strategic seller is $\pi^* = 900$ in each period.

In the next subsection, we introduce the speculator whose objective is to maximize his profit by buying in period 1 and selling in period 2 taking into account his storage costs.

II.B Duopoly Equilibrium with a Large Speculator

We now introduce the speculator who has access to a storage facility with fixed capacity, $\bar{q}_S$. The speculator buys in period 1, and stores it to sell in period 2 for a profit. As we will show, the speculator's actions link the prices in the two periods. First, we analyze the case where the disposal is prohibitively expensive; i.e., the speculator does not have the option to dispose of any portion of acquired inventory in period 2 when he submits his supply schedule (henceforth, *without disposal* case). Second, we consider a case where everything else is the same but the inventory disposal cost is zero (henceforth, *with disposal* case).

In both cases, the speculator chooses the limit buy price, p_S, and the limit buy quantity, $q_S \in [0, \bar{q}_S]$ to acquire inventory in period 1. When it comes to period 2, the speculator has to sell using a market order in the without disposal case, but can sell by using a combination of market order and stop-loss order in the with disposal case. In either case, the speculator takes action in period 1 knowing its impact on period 2 outcomes.

II.B.1 Duopoly Equilibrium with a Large Speculator, without Disposal

The speculator's buying strategy in period 1 will depend on the price he can get in period 2. We, therefore, start by analyzing the equilibrium in period 2 for all possible inventory levels of the speculator.

Period 2

We assume that the speculator starts period 2 with an inventory of q_S widgets. Since there is no disposal, he has to sell all the widgets in his inventory in the forward market at the beginning of the period for delivery at the end of the period. [8] Without loss of generality, we assume that the speculator sells using a market order in period 2.[9] Therefore, the market-clearing price in period 2 of the equilibrium without disposal is given by,

$$\boldsymbol{p}_2(Q_2) = a - b\,(Q_2 + q_S)\,, \tag{II.3}$$

where $Q_2 = q_{A2} + q_{B2}$ is the aggregate supply of the strategic sellers in period 2, q_S is the supply of the speculator. Each strategic seller decides how much she should supply, taking the aggregate demand schedule, the supply of the speculator, and the supply of the other strategic seller as given. In equilibrium, the aggregate demand of the consumers is equal to the aggregate supply of the strategic sellers plus the supply of the speculator, $Q_2 + q_S$.

As the speculator's supply, q_S, is fixed, the aggregate demand schedule in period 2 with the speculator equals the aggregate demand schedule in the benchmark case reduced by q_S units. The strategic sellers maximize their period 2 profit as follows:

$$\begin{aligned} &\max_{q_A} \boldsymbol{p}(q_A + q_B + q_S)\, q_A, \\ &\max_{q_B} \boldsymbol{p}(q_A + q_B + q_S)\, q_B, \end{aligned} \tag{II.4}$$

where $\boldsymbol{p}(q_A + q_B + q_S) = a - b(q_A + q_B + q_S)$.The best response functions of strategic sellers are:

$$\boldsymbol{q}_A(q_B) = \frac{a}{2\,b} - \frac{q_B}{2} - \frac{q_S}{2}, \quad \text{and} \quad \boldsymbol{q}_B(q_A) = \frac{a}{2\,b} - \frac{q_A}{2} - \frac{q_S}{2}. \tag{II.5}$$

The equilibrium supply of the strategic sellers as a function of the speculator's supply q_S is given below:

$$\boldsymbol{q}_2(q_S) = \frac{a - b\,q_S}{3\,b} = q^* - \frac{q_S}{3}, \tag{II.6}$$

where q_S is the speculator's supply in period 2. Compared to the benchmark supply, each strategic seller reduces supply by $\frac{q_S}{3}$ units so that reduction of both sellers taken together, $\frac{2}{3}\,q_S$, is less than the supply of the speculator, q_S. Therefore, in equilibrium the aggregate

[8] Although we assume zero storage losses, it is straightforward to incorporate such losses.

[9] Later, we show that the speculator profit-wise cannot do any better by using limit orders to sell his inventory in period 2 so long he is forced to liquidate the entire inventory in period 2.

supply is higher than the benchmark supply, q^*, and the market-clearing price is lower than the benchmark price, p^*.

$$\text{(II.7)} \qquad \boldsymbol{p}_2(q_S) = \frac{a - b\,q_S}{3} = p^* - \frac{b\,q_S}{3}.$$

To ensure that the price, $\boldsymbol{p}_2(q_S)$, and strategic sellers' supplies, $\boldsymbol{q}_2(q_S)$, are positive, we need an upper bound of the speculator's inventory $q_S < \frac{a}{b}$. This is a practical condition as it means although the speculator is quite large relatively to individual consumer, he is still small with respect to the aggregate market size. Then each strategic seller's profit in equilibrium in period 2 is

$$\text{(II.8)} \qquad \boldsymbol{\pi}_2(q_S) = \frac{(a - b\,q_S)^2}{9\,b} = \pi^* - \frac{q_S}{9}\,(2\,a - b\,q_S).$$

Compared to the benchmark equilibrium, each strategic seller supplies less as given in equation (II.6), and the price is strictly lower as shown in equation (II.7), and hence resulting profit of each strategic seller is lower in period 2 as shown in equation (II.8).

To see that the speculator cannot make higher profit by selling using a market order when the disposal is prohibitively costly, we consider the strategy where the speculator uses a limit order to supply his entire inventory but he chooses a limit price that is ϵ higher than the equilibrium period 2 price, $p^* - \frac{b\,q_S}{3}$. For such a price to be the equilibrium price, at least one strategic seller has to be better off by reducing her supply by $\frac{\epsilon}{b}$ units. This gives the deviating strategic seller a profit equal to $(q_2 - \frac{\epsilon}{b})(p_2 + \epsilon) = \frac{1}{b}\,(p_2^2 - \epsilon^2)$ which is lower than $\frac{1}{b}\,p_2^2$ which is the profit if she does not reduce her supply. Note that the speculator has to supply q_S units in period 2 so that he will comply with the market-clearing price in equation (II.7). This argument also applies to the case where the speculator supplies using a combination of market order and limit order since it still requires at least one strategic seller to reduce supply to achieve the speculator's limit price and if she does not have the incentive to reduce supply the equilibrium clearing price will remain unchanged.

Example 2:

Let the intercept, a, and slope, b, of the inverse demand function of the atomistic consumers, be the same as in example 1. Suppose that the speculator sells his inventory, $q_S = 15$, using a market order in period 2. The two solid lines in Figure 2 depict the best response functions of the strategic sellers in period 2 with the speculator. The equilibrium is given by the point where the two lines cross. From equation (II.6), it follows that the Cournot equilibrium supply of each strategic seller is given by: $q^* - \frac{q_S}{3} = 30 - \frac{15}{3} = 25$. The equilibrium aggregate supply, market-clearing price and profit of each strategic seller are 65, 25, and 625 respectively. Though each strategic seller produces less than the quantity in the benchmark equilibrium, the aggregate quantity supplied including the speculator's market order in period 2 (65 units) is greater than the aggregate benchmark supply (60 units), resulting in a lower price in period 2.

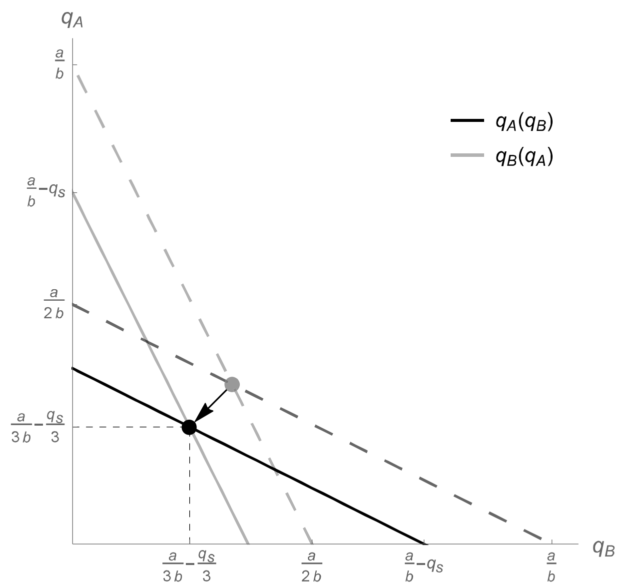

FIG. 2. Best response functions of the strategic sellers in period 2. The black and gray solid lines are the best response functions of strategic seller A and strategic seller B respectively when the speculator has no disposal option. Parameter vaues are $a = 90$, $b = 1$, and $q_S = 15$. The black and gray dashed lines represent the best response functions of strategic sellers in the benchmark case, and the gray dot (30, 30) denotes the aggregate equilibrium supply in period 2 (without the speculator). The black dot (25, 25) denotes the equilibrium supply in period 2 when the speculator cannot dispose of his inventory.

Period 1

In period 1, the speculator uses a limit order to acquire his inventory. Specifically, we assume that the speculator submits the following demand schedule

$$\boldsymbol{q}_S(p_1) = \begin{cases} 0 & \text{for } p_1 > p_S \\ [0,\ q_S] & \text{for } p_1 = p_S \\ q_S & \text{for } p_1 < p_S \end{cases} \tag{II.9}$$

where p_1 is the market-clearing price in period 1, and q_S is the quantity that the speculator buys when the clearing price is below the limit price p_S. When the clearing price is equal to p_S, the speculator accepts any partial execution. Each strategic seller maximizes the sum of her profits in the two periods, taking as given the supply of the other strategic seller, the limit order of the speculator, and the aggregate demand schedule of the consumers.

The equilibrium market price in period 1 of the equilibrium is a function of the aggregate supply of both strategic sellers, which in turn depends on the prices and quantities in the speculator's limit order, as given below:

$$\boldsymbol{p}_1(Q_1; q_S, p_S) = \begin{cases} a - b\,Q_1 & \text{for } Q_1 < \frac{a-p_S}{b} \\ p_S & \text{for } Q_1 \in \left[\frac{a-p_S}{b}, \frac{a-p_S}{b} + q_S\right] \\ a - b(Q_1 - q_S) & \text{for } Q_1 > \frac{a-p_S}{b} + q_S. \end{cases} \tag{II.10}$$

In equilibrium, the aggregate demand of the consumers in period 1 is equal to the aggregate supply of the strategic sellers minus the demand of the speculator. Figure 3 depicts the clearing price in period 1 of the equilibrium. If the strategic sellers find it is optimal to produce enough to meet the speculator's demand in full even though, this lowers the price, the speculator's limit order will be executed and the equilibrium price will be p_S.

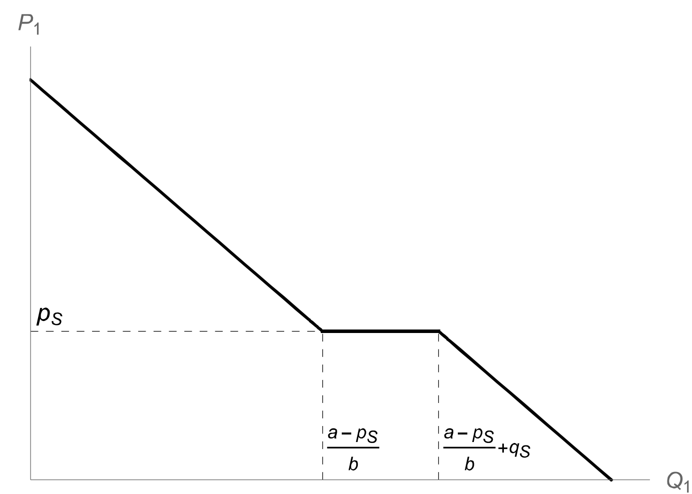

FIG. 3. Aggregate demand function and the speculator's limit order in period 1. This figure depicts the aggregate demand curve in period 1 when the speculator buys using a limit order. Parameters a and b are the intercept and the slope of the aggregate consumers' demand function. p_S and q_S are the limit buy price and limit buy quantity in the speculator's limit order. The horizontal segment of the demand curve indicates that the speculator accepts any partial execution of his limit order.

The objective function of each strategic seller is to maximize her profit in period 1 while taking into account the other strategic seller's supply decision, the speculator's demand in period 1, and supply in period 2. The two strategic sellers know that the speculator will have to dispose of in period 2, his entire inventory acquired in period 1. When this knowledge is taken into account, the period 1 objective functions of the two strategic sellers become:

$$\text{(II.11)}\qquad \begin{aligned}&\max_{q_{A1}} \boldsymbol{\pi}_{A1}(q_{A1}+q_{B1};q_S,p_S)+\boldsymbol{\pi}_2(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S))),\\&\max_{q_{B1}} \boldsymbol{\pi}_{B1}(q_{A1}+q_{B1};q_S,p_S)+\boldsymbol{\pi}_2(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S))),\end{aligned}$$

where

$$\text{(II.12)}\qquad \begin{aligned}&\boldsymbol{\pi}_{A1}(q_{A1}+q_{B1};q_S,p_S)=\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S)\,q_{A1},\\&\boldsymbol{\pi}_{B1}(q_{A1}+q_{B1};q_S,p_S)=\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S)q_{B1},\end{aligned}$$

and $\boldsymbol{\pi}_2(\cdot)$ is given by equation (II.8). Note that each strategic seller takes as given the demand schedule of the speculator, q_S, as a function of the price, in addition to the quantity supplied by the other strategic seller while making her optimal supply response.

The objective of the speculator is to maximize his trading profit, i.e., to maximize the difference between the cost of acquiring the inventory in period 1 and the revenue in period 2 net of the storage costs. We need the following two constraints to be satisfied: first, the participation constraint needs to hold; i.e., the trading profit of the speculator must be positive. Second, the incentive compatibility constraints of the strategic sellers need to be satisfied, i.e., it is in the interest of the strategic sellers to meet the speculator's demand. However, we do not have to explicitly impose the speculator's participation constraint since the speculator can choose $q_S = 0$ when per unit trading profit is negative. Therefore, the speculator chooses p_S and q_S to maximize his profit in (II.13) subjective to the incentive compatibility constraints of the two strategic sellers given in (II.14).

Formally, the speculator's objective can be written as follows:

$$\text{(II.13)}\qquad \max_{q_S,\,p_S}\quad \boldsymbol{q}_S\Big(\boldsymbol{p}_2(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S)))-\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S)-c_s\Big)$$

$$\text{(II.14)}\qquad \begin{aligned}\text{s.t.}\quad &\boldsymbol{\pi}_{A1}(q_{A1}+q_{B1};q_S,p_S)+\boldsymbol{\pi}_{A2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S))\big)\\&\qquad\geq \boldsymbol{\pi}_{A1}(q'_{A1}+q_{B1};q_S,p_S)+\boldsymbol{\pi}_{A2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q'_{A1}+q_{B1};q_S,p_S))\big)\\&\boldsymbol{\pi}_{B1}(q_{A1}+q_{B1};q_S,p_S)+\boldsymbol{\pi}_{B2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q_{B1};q_S,p_S))\big)\\&\qquad\geq \boldsymbol{\pi}_{B1}(q_{A1}+q'_{B1};q_S,p_S)+\boldsymbol{\pi}_{B2}\big(\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1}+q'_{B1};q_S,p_S))\big).\end{aligned}$$

In equation (3.14) above, the first inequality is the incentive compatibility constraint of strategic seller A, and the second is the incentive compatibility constraint of seller B. The incentive compatibility constraints restrict the choice of q_S and p_S in such a way that neither of the strategic sellers, i, has the incentive to supply q'_{i1} instead of q_{i1}, i = A, B.[10]

To derive the equilibrium we first note that the speculator can choose the limit order price and quantity in such a way that the incentive compatibility constraints of the strategic sellers given in (II.14)

[10]Our formulation of the problem follows the method in Kyle and Wang (1997).

hold as equalities.[11] We solve the model following the standard technique: we propose an equilibrium and show that none of the active agents has a unilateral incentive to deviate from the proposed equilibrium.

Given that the constraints (II.14) hold with strict equality and the fact that strategic sellers A and B adopt identical strategies on the equilibrium path, the speculator's choice reduces to choosing only one of the two choice variables: p_S or q_S but not both. This is because, in the symmetric equilibrium that we consider if one seller's incentive compatibility constraint is satisfied, the other seller's constraint will also be satisfied.

The speculator searches across potential price-quantity combinations corresponding to different limit orders and chooses the one that is the best for him. Without loss of generality, we use q_S as the choice variable of the speculator and let p_S be determined by the incentive compatibility constraints. We denote the optimal p_S for a given q_S that satisfies the incentive compatibility constraints as a function $\boldsymbol{p}_S(q_S)$. We then have the following lemma for the $\boldsymbol{p}_S(q_S)$.

Lemma 1. *Let p^* denote the benchmark equilibrium price as defined in equation* (II.2)*; q_S denote the limit buy-quantity in the limit order of the speculator; and a and b are the demand and sensitivity parameters respectively. Then, for any given q_S, there exists a limit buy price, $\boldsymbol{p}_S$ given by,*

$$\boldsymbol{p}_S(q_S) = p^* - \frac{1}{6}\sqrt{b\,q_S\,(4\,a + 13\,b\,q_S)} + \frac{b\,q_S}{2}, \tag{II.15}$$

such that p_S is also the market-clearing price and the speculator's demand q_S is executed in full.

Proof: See Appendix A.

To understand the economic intuition behind equation (II.15), first, note that the strategic sellers' incentive compatibility constraints have two implications for the speculator's position: at least one strategic seller needs the incentive to raise the supply to $(\frac{a-p_S}{b} + q_S - \frac{a}{3\,b})$ amount when the other strategic seller supplies the benchmark quantity of $(\frac{a}{3\,b})$ units; second, none of the strategic sellers has the unilateral incentive to deviate – i.e., to reduce or increase supply when the speculator's demand is met in full and equally supplied by the two strategic sellers, i.e., $\frac{1}{2}(\frac{a-p_S}{b} + q_S)$. The first implication leads to equation (II.15). The second implication is that when the speculator chooses any limit quantity $q_S < \frac{a}{b}$ and sets the limit buy price equal to $\boldsymbol{p}_S(q_S)$, then the market will be cleared at price $\boldsymbol{p}_S(q_S)$ in period 1. Note that equation (II.15) is not the necessary condition for the speculator's demand to be met in full. If the speculator sets the limit price lower than equation (II.15) while the second implication is satisfied, the speculator's limit price can still be the clearing price even though it is not unique since the benchmark price can also be the clearing price.

Although we allow for partial execution of the speculator's order in the model setup, it is also important to note that in equilibrium the speculator's demand q_S will always be fully supplied. Given

[11] This is equivalent to assuming that strategic sellers will supply enough to meet the speculator's demand if deviating and not deviating yield the same profit, and can be relaxed.

the speculator's demand is flat, the gain from supplying a greater portion of the speculator's demand in period 1 outweighs the corresponding share of the loss caused by additional supply in period 2 as the loss is equally shared by both strategic sellers and the speculator.

Example 3:

In this example, we provide the intuition behind Lemma 1. Let the values of the parameters, a and b, be the same as in example 2. Suppose that the speculator chooses a limit quantity, $q_S = 15$, and a limit price, $p_S = 22.3$. Table 1 shows the payoffs of the two strategic sellers given their supply decisions, "supply more" or supply the "benchmark" quantity. If one of the strategic sellers supplies 30 units, which is the same as what she supplied in the equilibrium with no speculator (Benchmark), the other strategic seller is indifferent between supplying 30 units and "supplying more". Together with the fact that no strategic seller would supply less when the other strategic seller supplies more than 30 units, both {Supply More, Supply More} and {Benchmark, Benchmark} are equilibria. However, if the speculator chooses a limit price $p_S = 22.3 + \epsilon$ where $\epsilon > 0$ and a limit quantity $q_S = 15$ and one of the strategic sellers supplies the Benchmark quantity, the other strategic seller will choose to "supply more" as the payoff goes up. In this case, {Supply More, Supply More} becomes the unique equilibrium which is depicted in Table 2.

TABLE 1

THE STRATEGIC SELLERS' SUPPLY AND PAYOFF
WHEN $Q_S = 15$ AND $P_S = 22.3$

		Seller B	
		Benchmark	Supply More
Seller A	Benchmark	$q^*_{A1}(=30), q^*_{B1}(=30)$	$q_{A1}(=30), q_{B1}(=52.7)$
	Supply More	$q_{A1}(=52.7), q_{B1}(=30)$	$q^{(*S)}_{A1}(=41.4), q^{(*S)}_{B1}(=41.4))$

		Seller B	
		Benchmark	Supply More
Seller A	Benchmark	$\pi^*_A(=1800), \pi^*_B(=1800)$	$\pi_A(=1294), \pi_B(=1800)$
	Supply More	$\pi_A(=1800), \pi_B(=1294)$	$\pi^{(*S)}_A(=1547), \pi^{(*S)}_B(=1547)$

NOTE. – These top and bottom tables report the supplies and the payoffs of each strategic sellers respectively when $q_S = 15$ and $p_S = 22.3$. Each strategic seller has two options: producing the benchmark quantity (Benchmark) or supplying for the speculator's limit order (Supply More). We rule out other options in the proof of Lemma 1 by showing that all other supply choices are inferior to these two choices. When $q_S = 15$ and $p_S = 22.3$, each strategic seller would be indifferent from "Benchmark" and "Supply More" when the rival sticks to "Benchmark" and therefore there will be two equilibria.

Suppose strategic seller B supplies 30 units and the speculator demands 15 units at a price of 22.3 per unit in period 1. The solid black line in the left (right) panel in Figure 4 gives the period-2 (period-

TABLE 2

THE STRATEGIC SELLERS' SUPPLY AND PAYOFF
WHEN $Q_S = 15$ AND $P_S = 22.3 + \epsilon$

		Seller B: Benchmark	Seller B: Supply More
Seller A	Benchmark	$q^*_{A1}(=30), q^*_{B1}(=30)$	$q_{A1}(=30), q_{B1}(=52.7)$
	Supply More	$q_{A1}(=52.7), q_{B1}(=30)$	$q^{(*S)}_{A1}(=41.4), q^{(*S)}_{B1}(=41.4))$

		Seller B: Benchmark	Seller B: Supply More
Seller A	Benchmark	$\pi^*_A(=1800), \pi^*_B(=1800)$	$\pi_A(=1294), \pi_B(=1800+\delta)$
	Supply More	$\pi_A(=1800+\delta), \pi_B(=1294)$	$\pi^{(*S)}_A(=1547), \pi^{(*S)}_B(=1547)$

NOTE. – These top and bottom tables report the supplies and the payoffs of each strategic seller respectively when $q_S = 15$ and $p_S = 22.3 + \epsilon$. Each strategic seller has two options: producing the benchmark quantity (Benchmark) or supplying for the speculator's limit order (Supply More). We rule out other options in the proof of Lemma 1 by showing that all other supply choices are inferior to these two choices. When $\epsilon > 0$, and therefore δ is greater than 0, "Supply More" when the other seller is supplying "Benchmark" dominates the "Benchmark". There will be only one equilibrium in which both strategic sellers choose to "Supply More".

1) demand curve faced by strategic seller A. The solid gray line in the left panel is the corresponding marginal revenue curve of strategic seller A when strategic seller B produces 25 units and the speculator supplies all 15 units of inventory using a market order. In this case, the best response of strategic seller A is to supply 25 units and the corresponding price is 25 denoted by the black dot in Figure 4. The dashed black and gray lines correspond to the hypothetical case where the speculator's limit order is not filled in period 1 and thus has nothing to supply in period 2 and strategic seller B supplies 30. Therefore, if strategic seller A decides to produce more to meet the speculator's limit order in period 1, her expected loss in period 2 relative to supplying the Benchmark quantity is the region of A-B-G-F-D-C-A.

When the speculator's demand is fulfilled in period 1, the increase in aggregate supply relative to Benchmark aggregate supply comes from two sources: the speculator's demand through the limit order and the increase in consumers' demand due to the price drop. This is why strategic seller A supplies 52.7 units, which is greater than the sum of the Benchmark supply of 30 units and the speculator's demand of 15 units. The strategic seller A will supply 52.7 units if she finds the period-1 gain (relative to supplying the Benchmark) of supplying more (region D-E-H-G-D minus region A-B-C-D-A in the right panel) is greater than the expected loss (relative to supplying the Benchmark) in period 2. Figure 5 further shows the sum of the two-period profits of strategic seller A as a function of her first period's response, when strategic seller B supplies 30 in period 1, and the Cournot equilibrium we characterized in Example 2 prevails in period 2.

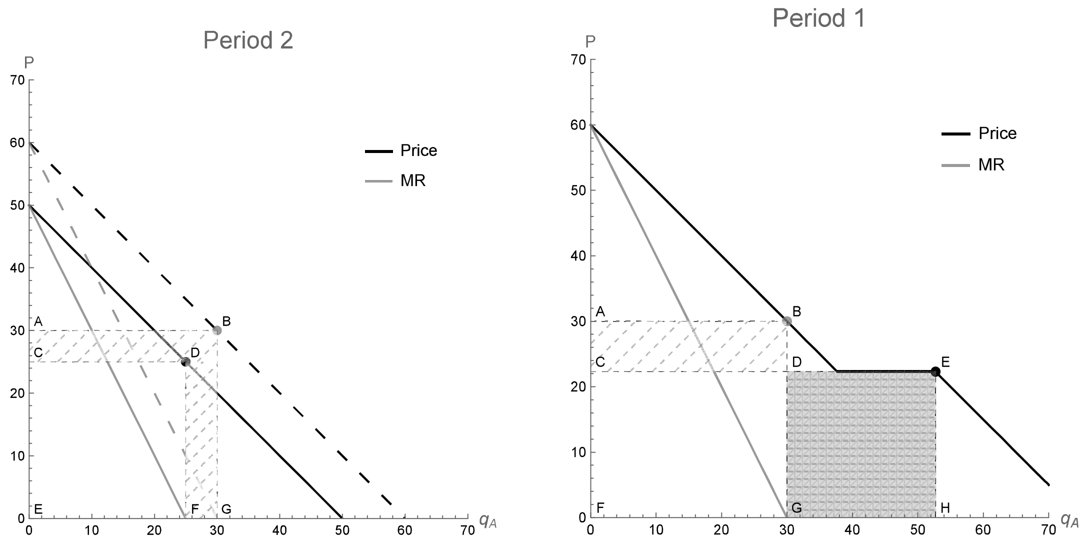

FIG. 4. Best response of strategic seller A in periods 1 and 2. This figure depicts the best responses of seller A for a given response of seller B.
Left panel depicts Period 2: The solid black line is the demand curve faced by strategic seller A when strategic seller B produces 25 units, and the speculator dumps 15 units. The solid gray line depicts the marginal revenue (MR) curve of strategic seller A (B). We assume that the marginal cost (MC) is zero. The black dot is the equilibrium with the speculator in period 2: A and B will produce 25 units. The dot-dashed black and gray lines correspond to the case without the speculator. The gray dot gives the equilibrium: A and B will produce 30 units. The gray color shaded area corresponds to the strategic sellers' profit when the speculator's limit order is fulfilled in period 1, and the area with tilted dashed lines corresponds to their profit in the benchmark case. Each strategic seller's payoff drops and the drop is equal to the area of region A-B-G-F-D-C-A.
Right panel depicts Period 1: The solid black line is the demand curve faced by the strategic seller A when the strategic seller B supplies 30 units the speculator submits a limit order to buy, 15 units at a price of 22.3 per unit. The black dot gives the equilibrium with the speculator in period 1: A produces 52.7 and B produces 30. The speculator buys 15 units. Although the black dot in the right panel is not a symmetric equilibrium, period 2's symmetric equilibrium corresponds to the black dot in the left panel where both supply 25 units and the speculator sells 15 units. The area of D-E-H-G and A-B-C-D correspond to the extra payoff and payoff drop of strategic seller A in period 1.
The gray dot represents the benchmark equilibrium where MR = MC = 0 (note that the gray dot is the equilibrium in period 1 implies that the gray dot has to be the equilibrium in period 2). When the area of D-E-H-G-D (right panel) is weakly greater than the area of A-B-C-D-A (right panel) and A-B-G-F-D-C-A (left panel), strategic seller A earns a profit weakly greater than the benchmark profit. This payoff structure eliminates the benchmark equilibrium as a potential equilibrium in the presence of the speculator.

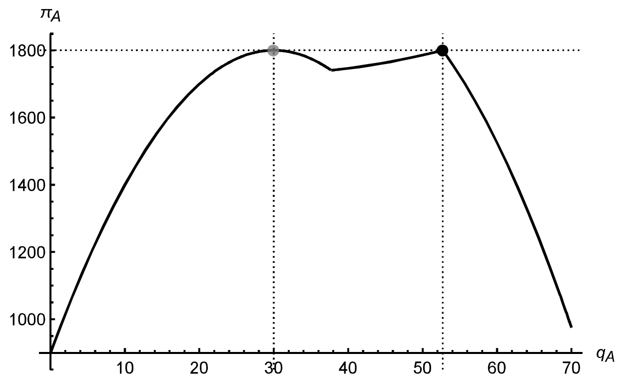

FIG. 5. Strategic seller A (B)'s payoffs. The vertical axis is the sum of the two-period profits of strategic seller A as a function of her first period response in the without disposal case, when strategic seller B supplies 30 in period 1. The gray and black dots correspond to the strategic seller A's supply choice in the benchmark case and in the without disposal case when the speculator demands 15 widgets using a limit order.

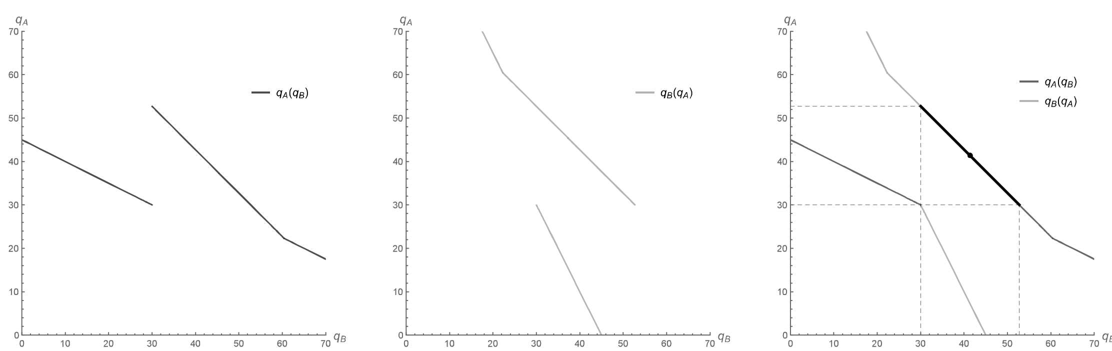

FIG. 6. Equilibrium in period 1. *Left panel*: The vertical axis gives the best response q_A of strategic seller A, when the supply of strategic seller B is q_B (on the x-axis), and the symmetric equilibrium with the speculator supplying his entire inventory of 15 units prevails in period 2. *Middle panel*: The horizontal axis gives the best response q_B of strategic seller B, when the supply of strategic seller A is q_A (on the y-axis), and symmetric equilibrium with the speculator supplying his entire inventory of 15 units prevails in period 2. *Right panel*: The two best response curves in the left and middle panel are superimposed to arrive at the equilibria. The two best response functions overlap on the black interval. The dark black interval depicts all equilibria, and the black dot ($q_A = 41.4, q_B = 41.4$) depicts the symmetric equilibrium.

By considering different values for the strategic seller B's supply, we plot the best response function of strategic seller A in the left panel of Figure 6. The vertical axis is the best supply q_A of strategic seller A when the supply of strategic seller B is q_B (on the x-axis). For $q_B \in [0, 30)$, the best response function of strategic seller A is $\boldsymbol{q}_A(q_B) = \frac{a}{2} - \frac{q_B}{2}$, with a slope of $-\frac{1}{2}$. For $q_B \in [30, 52.7)$, it is optimal for strategic seller A to supply enough to meet the speculator's limit demand, i.e., $\boldsymbol{q}_A(q_B) = a + q_S - p_S - q_B$, with a slope of -1. For $q_B \geq 52.7$, the best response function of strategic seller A becomes $\boldsymbol{q}_A(q_B) = \frac{a}{2} + \frac{q_S}{2} - \frac{q_B}{2}$, with a slope of $-\frac{1}{2}$. The horizontal axis of the middle panel of Figure 6 gives the best supply q_B of strategic seller B when the supply of strategic seller A is q_A (on the y-axis). In the right panel, we superimpose the two best response curves in the left and middle panels. The two best response functions overlap on the black interval and the black dot ($q_A = 41.4, q_B = 41.4$) is the only symmetric equilibrium. As can be seen, there are many asymmetric equilibria.

According to Lemma 1, if the speculator chooses the limit price as $\boldsymbol{p}_S(q_S)$ for a given q_S, the clearing price in period 1 will be $\boldsymbol{p}_S(q_S)$, i.e., $\boldsymbol{p}_1(q_{A1} + q_{B1}; q_S, \boldsymbol{p}_S(q_S)) = \boldsymbol{p}_S(q_S)$, and the speculator's demand q_S will be fully supplied, i.e., $\boldsymbol{q}_S(\boldsymbol{p}_1(q_{A1} + q_{B1}; q_S, \boldsymbol{p}_S(q_S))) = q_S$. Then, we can substitute out the incentive compatibility constraints of the strategic sellers in the speculator's optimization problem in equation (II.13) and represent it as follows:

$$\max_{q_S} \quad q_S\big(\boldsymbol{p}_2(q_S) - \boldsymbol{p}_S(q_S) - c_s\big). \tag{II.16}$$

We denote the quantity which solves the maximization in (II.16) as q_S^* and summarize the equilibrium without disposal in Proposition 1.

Proposition 1. *When the storage cost is lower than $\bar{c}_S$, where*

$$\bar{c}_S = \frac{5 - 2\sqrt{3}}{39}\, a \approx 0.04\, a,$$

the speculator submits a limit order in period 1 where the limit price is given in Lemma 1 and the limit quantity is the minimum between his fixed storage capacity, $\bar{q}_S$ and the profit-maximizing quantity, q_S^* which solves the maximization problem in expression (II.16)*, i.e., $q_S = \min\{\bar{q}_S, q_S^*\}$, and sells all of his acquired inventory in period 2 using a market order. There exists an equilibrium where the period 1 clearing price is equal to the speculator's limit buy price and the period 2 clearing price is equal to $p^* - \frac{b\, q_S}{3}$ given in equation* (II.7)*, and the speculator makes a positive trading profit. When the storage cost is greater than $\bar{c}_S$, the speculator does not enter the markets in any of the two periods.*

Proof: See Appendix A.

Note that the equilibrium prices in period 1 and period 2 are different when there is a speculator, i.e., speculation creates price volatility. In period 2 of the equilibrium without disposal, the speculator is a price-taker so the clearing price is determined by the competition of the two strategic sellers. In period 1, the speculator, however, submits a limit order which gives the two strategic sellers an incentive to supply the quantity he demands while lowering the clearing price to make a profit. In equilibrium, the period 1 price is lower than the period 2 price, and as long as the maximum price spread the speculator can generate is greater than his inventory cost per unit, he is able to make a positive trading profit. Note that if the unit net price, $(p_2 - p_1 - c_S)$ is negative, the speculator will not trade, i.e., $q_S = 0$.

The size of the speculator's capacity also plays a critical role in price spread and the speculator's profit. We solve for the speculator's limit buy-quantity which maximizes the price spread and denote it $\hat{q}_S$: $\hat{q}_S = \frac{(5\sqrt{3}-6)a}{39\,b} = \frac{2.66\,a}{39\,b}$. Then, we solve for the speculator's limit buy-quantity which maximizes his profit, and denote it q_S^*: $q_S^* = \frac{9\,a}{(11+5\sqrt{61})b} = \frac{9\,a}{50.05\,b} < \hat{q}_S$. Therefore, the price spread may be greater than the storage capacity is limited below q_S^*.

Example 4:

Let the parameters, a and b, be the same as in Example 3. Suppose that the speculator's storage cost is $c_S = 0.5$. When $c_S = 0.5$, the optimal inventory acquired by the speculator in period 1 will be 15 as given in Example 3. The left (right) panel of Figure 7 plots the price spread, $p_2 - p_1$, (speculator's profit, π_S) as a function of the limit buy-quantity of the speculator. The solid (dashed) line in the right panel in Figure 7 shows the speculator's profit when $c_S = 0.5$ ($c_S = 2.0$). The speculator's profit is maximized when his limit buy-quantity is 15 (11.1) and the resulting price spread $(p_2 - p_1)$ is 2.7 (3.2).

The left panel of Figure 7 shows that the price spread reaches the maximum when the storage capacity is $\hat{q}_S = 6.1$ units.[12] Suppose the storage capacity is 6 units. Then it can be shown that the speculator's optimal limit buy-quantity will be 6 units, and the limit price will be 24.5 (see equation (II.15)). The speculator will sell all 6 units in period 2 using a market order. The resulting market-clearing price in period 2 will be 28, i.e., a price spread of $p_2 - p_1 = 3.5$ and a volatility of 11.8%.

The following corollary summarizes the impact of the speculator's storage capacity on the price volatility and the consumer surplus.

Corollary 1. *In the case without disposal, the equilibrium market-clearing prices in period 1 and period 2 are both below the benchmark equilibrium price; thus, the consumers are better off in both periods. Both strategic sellers are worse off. The speculator's trading profit is less than the aggregate loss of the strategic sellers in periods 1 and 2 taken together, even without storage cost. The price spread is increasing at first with storage capacity, reaching its maximum at $\hat{q}_S$ units, and then strictly decreasing thereafter. The speculator's profit is maximized at q_S^* which is strictly greater than the price spread maximizing capacity.*

Proof: See Appendix A.

[12] Although the price spread is maximized at 6.1 units, we assume the nearest integer value of 6 for storage capacity for expositional simplicity.

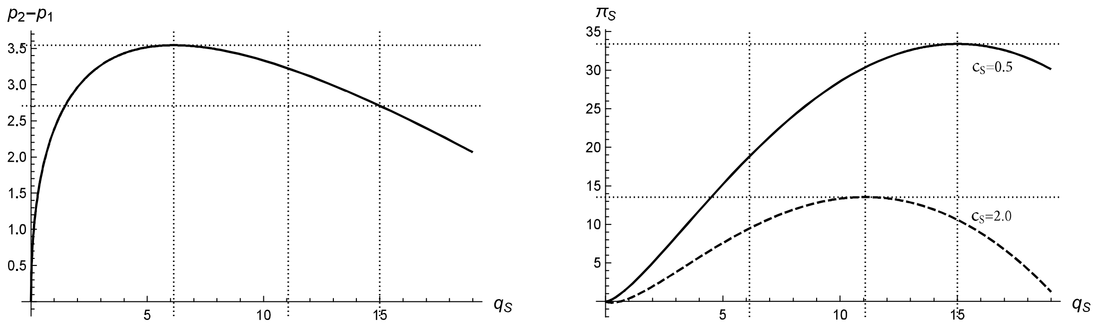

FIG. 7. Price volatility and the speculator's profit as a function of limit buy quantity. The left panel of the figure depicts the relationship between price spread and the size of the acquired inventory, q_S, in the without disposal case. The right panel of the figure depicts the relationship between the trading profit, π_S, of the speculator and the size of the acquired inventory with two different per-unit storage costs. We assume the parameters are $a = 90$ and $b = 1$, and the per-unit storage costs are $c_S = 0.5$ (denoted by the solid line) and $c_S = 2.0$ (denoted by the dashed line). The price spread, $p_2 - p_1$, reaches its maximum value 3.5 if the speculator chooses $\hat{q}_S = 6.1$, while the speculator earns the highest profit $\pi_S = 33.4$ (13.5) dollars if he chooses $q_S = 15$ (11.1) when the per-unit storage cost is $c_S = 0.5$ (2.0).

The strategic sellers are always worse off because the speculator sets his limit price in a way such that the strategic seller who unilaterally deviates to supply more in period 1 earns just the benchmark profit. This leaves the strategic seller who does not increase supply in period 1 a lower profit as she sticks to the benchmark quantity and the market-clearing price in period 1 is lower than the benchmark price. Together with the fact that both strategic sellers earn equal profit in period 2, the strategic seller who does not change supply is strictly worse off compared to her rival. Moreover, the loss of the strategic seller who does not change supply is greater than the speculator's trading gain. However, the lower prices in both periods make the consumers better off.

II.B.2 Duopoly Equilibrium with a Large Speculator, and free Disposal

In this section, we assume that the speculator can dispose of his unsold inventory at the end of period 2 without incurring any cost. In this case, the speculator will use a combination of market order and stop-loss order in period 2 to sell some of his inventory of widgets acquired using a limit order in period 1 and dispose of the rest of the widgets without selling them. The speculator optimally chooses four values: the limit buy price in period 1, p_{S1}, the limit quantity in period 1, q_{S1}, the stop-loss price in period 2, p_{S2}, and the stop-loss quantity, q_{S2}. In what follows, we characterize the equilibrium market-clearing prices, profits earned by the speculator and the strategic sellers, and consumers' welfare.

Period 2

Suppose the speculator comes into period 2 with an inventory of q_{S1}, which he bought in period 1, and submits two orders to supply: a market order to supply $\alpha\, q_{S1}$, $\alpha \in [0, 1]$, and a stop-loss order to supply $(1-\alpha)\, q_{S1}$ when the clearing price in period 2 is below p_{S2} and zero otherwise. We assume that the speculator can dispose of $(1-\alpha)q_{S1}$ units of widgets without incurring any costs when the clearing price is above or equal to p_{S2}.[13] Taking the two orders together, we get the following supply schedule of the speculator given below:

$$\text{(II.17)} \qquad \boldsymbol{q}_{S2}(p_2) = \begin{cases} \alpha\, q_{S1} & \text{for } p_2 \geq p_{S2} \\ q_{S1} & \text{for } p_2 < p_{S2} \end{cases}$$

where p_2 is the clearing price in period 2, p_{S2} is the stop-loss price the speculator chooses, and α is between 0 and 1.[14] The supply schedule in (II.17) implies that the speculator is willing to "sacrifice" a fraction, $1-\alpha$, of his inventory when the clearing price in period 2 is equal to or above the stop-loss price p_{S2}. If the price is below p_{S2}, the speculator supplies his entire inventory q_{S1}.

Taking the speculator's supply schedule into account, the clearing price in period 2 of the equilibrium with free disposal is given by:

$$\text{(II.18)} \qquad \boldsymbol{p}_2(Q_2; \alpha, p_{S2}, q_{S1}) = \begin{cases} a - b(Q_2 + \alpha q_{S1}) & \text{for } Q_2 \leq \frac{a-p_{S2}}{b} - \alpha q_{S1} \\ a - b(Q_2 + q_{S1}) & \text{for } Q_2 > \frac{a-p_{S2}}{b} - \alpha q_{S1} \end{cases}$$

where $Q_2 = q_{A_2} + q_{B_2}$ is the aggregate supply of the two strategic sellers in period 2. In equilibrium, the aggregate quantity supplied depends on the parameters, a and b, of the consumers' aggregate demand function; the fraction α of the inventory that the speculator acquired in period 1 which he supplies in period 2 using a market order; and the trigger price p_{S2} of the stop-loss order that the speculator chooses for selling the remaining $1-\alpha$ fraction of his inventory. If the speculator chooses $\alpha = 1$, i.e., disposes of his inventory using a market order, the clearing price in period 2 will be the same as the equilibrium price in the economy without disposal.

The leftmost panel of Figure 8 depicts the aggregate demand function of atomistic consumers as assumed in the benchmark case. The middle panel of figure 8 depicts the speculator's supply schedule based on a combination of stop-loss and market order. The rightmost panel of Figure 8 can be interpreted as the net demand function that the strategic sellers face while taking their individual supply decision. We obtain the net demand function by subtracting the speculator's strategic supply schedule from the aggregate demand schedule of the consumers.

The strategic sellers maximize their profits in period 2 by taking into account the speculator's market

[13]All the results in the paper will go through when the cost of disposal is relatively small. For example, the speculator may be able to transport it to another market and sell it at a small loss.

[14]We add subscripts 1 and 2 to q_{S1} to differentiate the speculator's quantity demand in period 1 and quantity supply in period 2. We also add subscripts 1 and 2 to p_S to differentiate the limit prices the speculator chooses in period 1 and period 2.

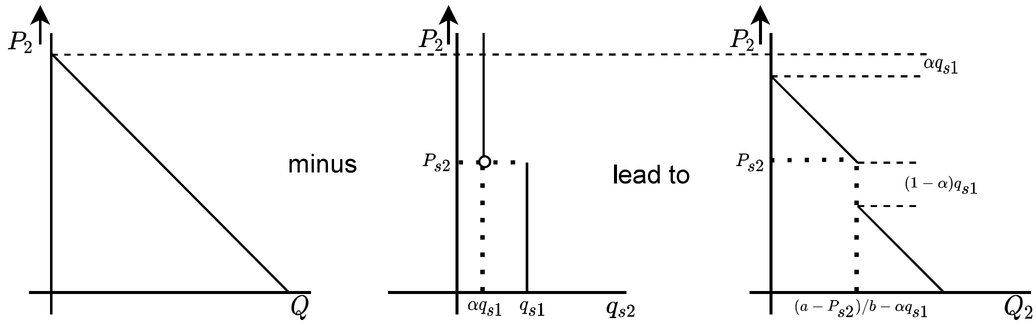

FIG. 8. Speculator's supply in period 2 with free disposal. The figure depicts the speculator's period 2 supply function using a combination of market order and stop-loss order. The speculator supplies a fraction α of his acquired inventory in period 1, q_{S1}, using a market order. The remaining fraction $1-\alpha$ of his acquired inventory is supplied using a stop-loss function with a trigger price, p_{S2}. This figure depicts the clearing price in period 2 equilibrium with free disposal where Q_2 denotes the total supply of the strategic sellers. When the aggregate supply of the strategic sellers is greater than $\frac{a-p_{S2}}{b}-\alpha\, q_{S1}$ price drops below p_{S2}. That kicks an additional supply of $(1-\alpha)\, q_{S1}$ from the speculator causing the price to drop even further.

and stop-loss orders, the supply of the other strategic seller, and the demand curve, i.e.,

$$\text{(II.19)}\qquad \begin{aligned} &\max_{q_{A2}} q_{A2}\boldsymbol{p}_2(q_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1}),\\ &\max_{q_{B2}} q_{B2}\boldsymbol{p}_2(q_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1}),\end{aligned}$$

The speculator's objective is to maximize his revenue in period 2 when the incentive compatibility constraints of the strategic sellers are satisfied, i.e.,

$$\text{(II.20)}\qquad \begin{aligned} \max_{\alpha,p_{S2}}\quad & \boldsymbol{q}_{S2}(q_{S1})\boldsymbol{p}_2(q_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1})\\ \text{s.t.}\quad & q_{A2}\boldsymbol{p}_2(q_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1})\geq q_{A2}\boldsymbol{p}_2(q'_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1})\\ & q_{B2}\boldsymbol{p}_2(q_{A_2}+q_{B_2};\alpha,p_{S2},q_{S1})\geq q_{A2}\boldsymbol{p}_2(q_{A_2}+q'_{B_2};\alpha,p_{S2},q_{S1})\end{aligned}$$

where q'_{A2} and q'_{B2} denote any alternative supply strategies of the strategic seller A and strategic seller B in period 2 respectively. We assume that the speculator chooses the limit price p_{S2} at the level which makes the strategic sellers just be indifferent from deviating and not deviating. The following lemma summarizes the speculator's strategy in period 2 of the equilibrium with free disposal.

Lemma 2. *For any given level of inventory q_{S1}, the speculator supplies α fraction of his inventory using a market order and $(1-\alpha)$ fraction of his inventory using a stop-loss order when the price falls below p_{S2}. α and p_{S2} as functions of q_{S1} are given below:*

$$\text{(II.21)}\qquad \boldsymbol{\alpha}(q_{S1})=\frac{3\,a}{2\,a+b\,q_{S1}}-\frac{1}{2}$$

$$\boldsymbol{p}_{S2}(q_{S1}) = p^* + \frac{b\,q_{S1}}{6} \tag{II.22}$$

where $p^* = \frac{a}{3}$ is the benchmark price. The stop-loss order's price-quantity pair, $(\boldsymbol{p}_{S2}, (1 - \boldsymbol{\alpha}(q_{S1}))\,q_{S1})$ will be such that the period 2 clearing price p_2 will be equal to p_{S2} and the stop-loss order will not be executed. The speculator gains $\boldsymbol{\alpha}(q_{S1})q_{S1}p_2$ and is better off when compared to the equilibrium without disposal.

Proof: See Appendix A.

Notice that the stop-loss order has an interesting feature: the speculator's supply increases when the price drops. This is the opposite of what happens in classical competitive markets where supply comes down as the price comes down. The stop-loss order makes the market price crash when the strategic sellers' combined supply is large enough to trigger the execution of the stop-loss order. Therefore, it is in the interest of the strategic sellers to restrict the amount they supply to ensure that the stop-loss order will not be executed. As is evident from equation (II.22), in equilibrium, the speculator chooses a stop-loss price that is strictly greater than the benchmark price in the economy without speculator.

For any given α and q_{S1}, the incentive compatibility constraints in equation (II.20) impose an upper bound on the stop-loss price p_{S2} that the speculator can set. However, the market-clearing price in period 2 as given in equation (II.22) is a function of only q_{S1} because the speculator chooses the optimal α to maximize the revenue taking into account of the functional relation between p_{S2} and (α, q_{S1}).

The optimal fraction for the speculator to sell using a market order, α, is a decreasing function of the inventory q_{S1}. The speculator can set a higher incentive-compatible stop-loss price when he has a larger inventory since the execution of such a stop-loss order becomes a bigger "threat" to the strategic sellers. Even though he sells a smaller fraction of his inventory, the total quantity he sells is higher resulting in higher revenue.

In equilibrium, the two strategic sellers together supply a smaller number of widgets in period 2 when compared to the economy without disposal to avoid the execution of the stop-loss order. The stop-loss order ensures that one strategic seller will profit more by reducing supply when the other strategic seller supplies what was the equilibrium quantity in period 2 in the economy without disposal. In equilibrium, both strategic sellers will supply less than the quantities supplied in the economy without disposal in period 2. The corollary 2 summarizes the profits of the two strategic sellers and the welfare of the consumers in period 2 of the equilibrium with disposal.

Corollary 2. *When free disposal is available, there is a period 2 equilibrium where both strategic sellers reduce their supplies by the same amount when compared to their period 2 equilibrium supplies in the case without disposal. The period 2 profit of each strategic seller is given by*

$$\pi^*_{A2}(or\ \pi^*_{B2}) = \frac{(4a - b\,q_{S1})(a - b\,q_{S1})}{36\,b},$$

which is strictly greater than the period 2 profit in the equilibrium without disposal. There is also an asymmetric period 2 equilibrium where one strategic seller supplies the same amount as in the equilibrium without disposal, and the other strategic seller supplies less to avoid the execution of the speculator's stop-loss order. In this asymmetric equilibrium, the strategic seller who supplies less earns a higher profit. In any period 2 equilibrium the market-clearing price in period 2, p_{S2} is higher than the market-clearing price, p^* in the benchmark case. Consumers are worse off in period 2 when compared to the benchmark case without the speculator.

We will appeal to Corollary 2 when we solve for the speculator's optimal strategy in period 1. The speculator takes into account how his combination of stop-loss and market orders affect the equilibrium in period 2.

Example 5:

Let the parameter values, $a = 90$, $b = 1$, and $q_{S1} = 15$, be the same as in Example 2. The left panel of Figure 9 depicts the period 2 clearing price when the speculator chooses an α and a stop-loss price given by equation (II.22). As the fraction to supply through stop-loss order, $1 - \alpha$ gets bigger, the clearing price in period 2 becomes higher. Recall that the benchmark clearing price (in the economy without the speculator) is given by $p^* = 30$, which is lower than $\boldsymbol{p}_{S2}(q_{S1})$ when $\alpha > 0.056$, and the period 2 clearing price in equilibrium without disposal is 25, which is lower than $\boldsymbol{p}_{S2}(q_{S1})$ as long as $1 - \alpha > 0$. The speculator optimally chooses to dispose of 12% of his acquired inventory; i.e., $1 - \alpha = 0.12$ which results in a market-clearing price of $p_2 = 32.5$ and the speculator's revenue is equal to $q_{S2}\,p_2 = 429$ in period 2. We show that the incentive compatibility constraints of both strategic sellers are satisfied. The benchmark market-clearing price is 30 which is lower than p_2, implying that the consumers are worse off in period 2.

In the equilibrium without disposal, the speculator earns $25 \times 15 = 375$ in period 2 by supplying his entire inventory for sale. In the equilibrium with disposal, the speculator earns more than 375 in period 2 as long as he disposes of less than 37% of his total acquired inventory; i.e., $(1 - \alpha) \leqslant 0.37$. This is depicted on the right panel of Figure 9. In one of the many asymmetric equilibria, one of the strategic sellers will supply 19.3 units with a period 2 profit of 625 (which is equal to the period 2 profit in the equilibrium without disposal), and the other strategic seller supplies 25 units (the same as the equilibrium supply in period 2 when there is no disposal) with a period 2 profit of 812.5.

In the symmetric equilibrium, both strategic sellers will reduce their supplies equally to avoid the execution of the speculator's stop-loss order. It can be shown that each strategic seller will supply $q_2 = 22.12$ units. This results in a period 2 profit of $q_{S2}\,p_2 = 718.9$ for each strategic seller. The speculator's stop-loss order will get filled if any one of the strategic sellers increases her supply.

Period 1

In period 1, the speculator maximizes his profit by choosing the price p_{S1} and quantity q_{S1} for his

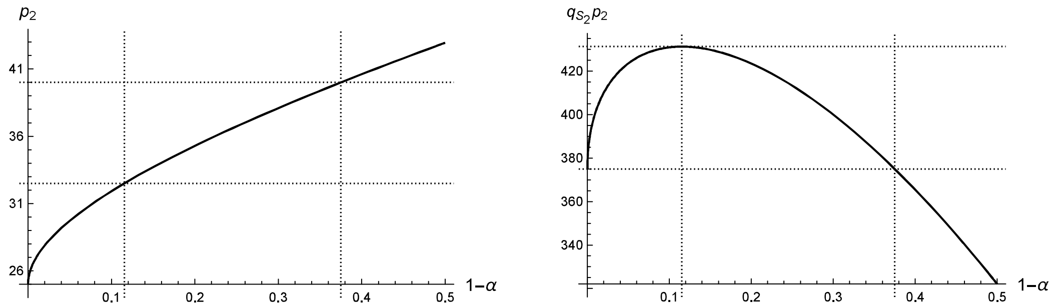

FIG. 9. Period 2 market-clearing price and speculator's profit vs fraction supplied using market order. This figure depicts the period 2 clearing price (on the left panel) and the period 2 profit of the speculator (on the right panel) according to different values of $\alpha \in [0.5, 1.0]$ with parameters $a = 90$, $b = 1$, and $q_{S1} = 15$. Both panels include the highest limit price that satisfies the incentive compatibility constraints and the highest limit price which guarantees that it is the unique clearing price in period 2.

limit order taking into account how it will affect the equilibrium in period 2. The demand schedule is given in equation (II.9) in the equilibrium without disposal where q_S and p_S should be replaced by q_{S1} and p_{S1} respectively. Applying the argument we used in the case without disposal, we get the following relation between p_{S1} as a function of q_{S1}:

$$\text{(II.23)} \qquad \boldsymbol{p}_{S1}(q_{S1}) = p^* - \frac{1}{6}\sqrt{b\,q_{S1}\,(4\,a + 13\,b\,q_{S1})} + \frac{b\,q_{S1}}{2}.$$

The above relationship between p_{S1} and q_{S1} ensures that the incentive compatibility constraints given in (II.14) are satisfied. This is because the lowest profit a strategic seller earns when the two periods are taken together is the same as the equilibrium profit in the economy without disposal which as we already showed satisfies the incentive compatibility constraints. In equilibrium, the speculator has to choose only the optimal q_{S1} to maximize his trading profit taking into account the storage cost which is given by:

$$\text{(II.24)} \qquad \max_{q_{S1}} \quad q_{S1}\Big(\boldsymbol{\alpha}(q_{S1})\boldsymbol{p}_2(q_{S1}) - \boldsymbol{p}_{S1}(q_{S1}) - c_S\Big).$$

The equilibrium is summarized by the following proposition.

Proposition 2. *There is an equilibrium where the per unit storage cost is less than the per unit profit of the speculator, i.e.,*

$$\text{(II.25)} \qquad c_S \leq \boldsymbol{\alpha}(\bar{q}_S)\boldsymbol{p}_2(\bar{q}_S) - \boldsymbol{p}_{S1}(\bar{q}_S).$$

The speculator will buy in period 1 using a limit order where the limit quantity is his maximum storage capacity, $\bar{q}_S$, and the limit price is given by equation (II.23)*. He will sell in period 2 using the combination* *of stop-loss and market orders where the stop-loss price and quantity are given in Lemma 2. The period 1 market clearing price is equal to the speculator's limit buy price and the period 2 market clearing price is equal to the speculator's stop-loss order price.*

Proof: See Appendix A.

The speculator's trading profit is $q_{S1}(\alpha p_{S2} - p_{S1})$ which can be thought of as quantity times a weighted price spread as he disposes of the $(1-\alpha)$ fraction of his inventory in period 2. While the weighted period 2 price, $\alpha p_{S2} = p^* - \frac{b\,q_{S1}}{12}$, decreases with q_{S1}, the period 1 price p_{S1} decreases faster. This makes the speculator always fully utilize his capacity constraint when free disposal is available.

As the period 2 market clearing price, p_{S2}, is an increasing function of q_{S1} and period 1 market clearing price, p_{S1}, is a decreasing function of q_{S1}, the price spread will increase when the speculator demands more. Let $w^* = \frac{1}{b}(a-p^*)^2$ denote the consumer surplus and $w^{*s} = \frac{1}{2b}(a-p_1)^2 + \frac{1}{2b}(a-p_2)^2$ denote the sum of the consumer surpluses in period 1 and period 2 in the equilibrium with the speculator. It can be verified that consumers are better off when both period 1 and period 2 are taken together, while they can be worse off in period 2. Since consumers are myopic and price takers in our model, they will complain about high prices due to speculation in period 2. In the next section we generalize the model to an economy with more than two strategic sellers and show that consumers can be worse off even when the two periods are taken together.

Example 6:

Let $a = 90$, $b = 1$, and storage cost $c_S = 0.5$ as in the earlier examples. Recall that we showed in Example 5 that the equilibrium clearing price in period 2 in the economy with free disposal will be greater than the equilibrium price in the benchmark economy with no speculator. We now assume that the speculator's maximum storage capacity, $\bar{q}_S$, is 15 units and derive the speculator's optimal limit order in period 1. In equilibrium, in period 1, the speculator will buy using a limit order with a limit price of $p_{S1} = 22.3$ and a limit quantity of $q_{S1} = 15$. In equilibrium, in period 2, the speculator will sell his inventory using two types of orders: (1) a market order to sell $\alpha\, q_{S1} = 0.88 \times 15 = 13.2$ units, and (2) a stop-loss order to sell $(1-\alpha)\, q_{S1} = 0.12 \times 15 = 1.8$ when the price drops strictly below $p_{S2} = 32.5$. In equilibrium, the stop-loss order will not be executed and the market clearing price will be $p_{S2} = 32.5$. The speculator will be left with α fraction of his unsold inventory which he will dispose of at no cost. The speculator's profit in the economy with disposal will be $\alpha\, q_{S1}\, p_2 = 429$, which is greater than the profit of 375 in the economy without disposal.

Corollary 3. *The following properties hold in any equilibrium:*

i. The speculator's profit and the spread between the prices in the two periods are increasing functions of the speculator's storage capacity $\bar{q}_S$.

ii. Consumers are worse off in period 2 when compared to the equilibrium in the benchmark economy without the speculator. However, they are better off when the two periods are taken together.

iii. The strategic sellers are better off when compared to the equilibrium in the benchmark economy without the speculator.

In the economies we considered, profitable destabilizing speculation requires the presence of strategic sellers. Speculation will not be profitable if the sellers as well as the consumers are price takers. A natural question that arises is whether speculative profit will decline as the number of strategic sellers increases. We examine this question in the next section.

III Model Extensions

III.A Oligopolistic Sellers and the Welfare of Consumers

In this section, we extend the analysis to an economy with one large speculator and many oligopolistic/strategic sellers. Since the analysis closely follows the analysis of the equilibrium with two strategic sellers, we provide the equations characterizing the equilibrium in Appendix B. By making use of the equations, we show that consumers can be worse off in equilibrium relative to the equilibrium in the benchmark economy without the speculator, in the numerical example below.

Example 7:

Suppose that there are $m = 15$ strategic sellers and the parameters be the same as in earlier examples. It can be verified that in the benchmark economy without the speculator, the market clearing price is 5.6. Each strategic seller will supply 5.6, i.e., the aggregate supply is $15 \times 5.6 = 84$, and earn a profit of 31.4 consistent with equation (B.1) in Appendix B. As in the duopoly case, the speculator will fully utilize his storage capacity to maximize his trading profit, and accumulate an inventory $q_{S1} = 15$ units in period 1. In the economy without disposal, the speculator will sell all his inventory using a market order and each strategic seller will supply 4.7 units in period 2. The equilibrium period 2 market clearing price will be 4.7. Each strategic seller will earn a profit of 22 when the two periods are taken together. When there is free disposal, the speculator optimally chooses to supply 0.69 of his inventory through a market order and the rest through a stop-loss order with a price of 12.2. To prevent the execution of the stop-loss order, each strategic seller will reduce her supply to 1.8 units resulting in a profit of 22. Note that the speculator's revenue in period 2 is 70.3 when he sells using a market order and is 126.6 when he sells using the combination of a market order and a stop-loss order.

In the equilibrium with free disposal, the period 1 clearing price is 1.7 and the period 2 clearing price is 12.2 which leads to a consumer surplus of 3900 in period 1 and 3027 in period 2 which is lower than the benchmark equilibrium in which the consumer surplus is 3560 in both periods. We use the same parameters that we used in this example and solve it for a range of $m \in [2, 20]$. Figure 10 plots the consumer surplus of the two periods, the clearing price in the benchmark equilibrium, and the period 1 and period 2 clearing prices in the equilibrium with free disposal.

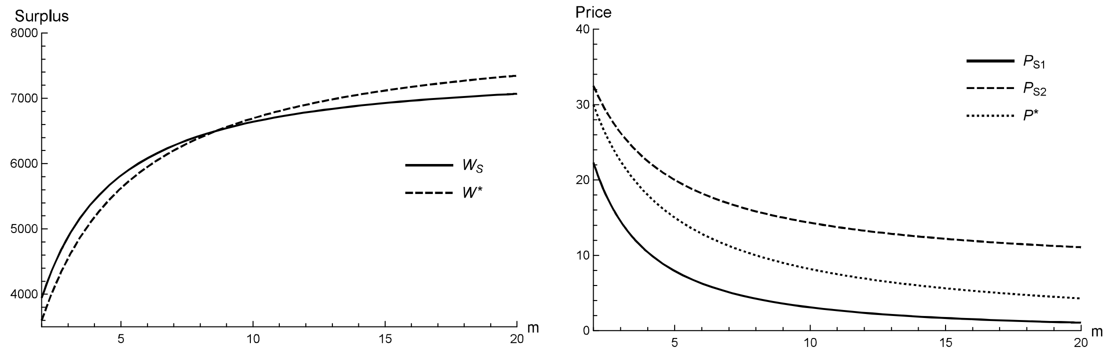

FIG. 10. Consumers' surplus and the market-clearing price with more than two sellers. This figure depicts the consumer surplus (on the left panel) and the clearing prices (on the right panel) according to a range of $m \in [2, 20]$ with parameters $a = 90$, $b = 1$, and $\bar{q}_S = 15$. W^* and W_S denote the consumer surplus in the benchmark equilibrium and the equilibrium with free disposal respectively, and p^*, p_{S1}, and p_{S2} denote the clearing price in the benchmark equilibrium and period 1 price and period 2 price in the equilibrium with free disposal respectively.

From the left panel of Figure 10, we can see that the consumers will be worse off when $m \geq 9$ and there is free disposal when compared to the benchmark equilibrium. The right panel of Figure 10 explains why speculation lowers consumer surplus when there are more strategic sellers. The equilibrium prices in all the economies decrease as m increases. As the number of strategic sellers increases, the difference between the period 1 price with free disposal and the benchmark price in the economy without the speculator decreases, which is good for the consumers. However, the difference between the period 2 price with free disposal and the benchmark price in the economy without the speculator increases faster. Since consumer's surplus is given by $w^{*s} = \frac{1}{2b}(a - p_1)^2 + \frac{1}{2b}(a - p_2)^2$, speculation can make consumers worse off when there are more strategic sellers.

In Figure 11, the consumers are better off in the equilibrium with free disposal than in the benchmark equilibrium when either m or $\bar{q}_S$ is small. This is because when either m or $\bar{q}_S$ is small, the "bargaining" power of the speculator relative to an individual strategic seller is small in period 2 so that the period 2 clearing price is not high enough to outweigh the benefit of the low clearing price in period 1. When either m or $\bar{q}_S$ increases, however, the speculator's inventory becomes a bigger "threat" to any individual strategic seller. In other words, the highest stop-loss price which satisfies the incentive compatibility constraints is higher when the size of an individual strategic seller becomes smaller or the speculator carries more inventory into period 2.

Equation (B.8) in the Appendix B gives the maximum per-unit profit of the speculator, $\alpha p_{S2} - p_{S1}$, when there are m strategic sellers. As the speculator always acquires the maximum inventory he can take

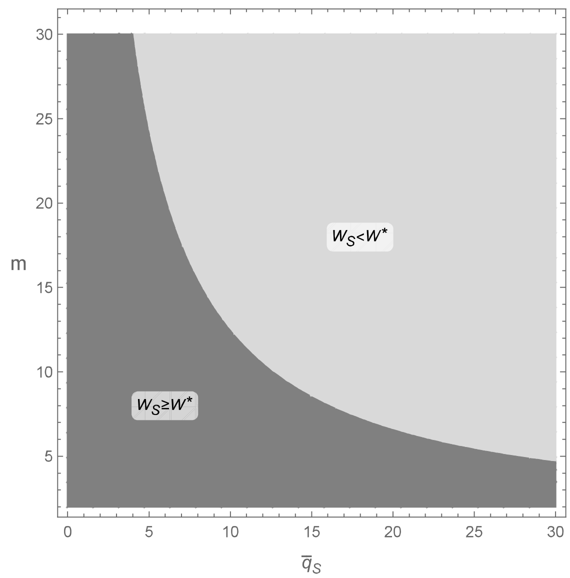

FIG. 11. Impact of the number of sellers, m, and the speculator's storage capacity, $\bar{q}_S$, on consumers' surplus. This figure plots the region where the consumer surplus is greater (less) in the equilibrium with free disposal than in the benchmark equilibrium for $m \in [2, 30]$ and $\bar{q}_S \in [0, 30]$ with $a = 90$ and $b = 1$. The dark gray (light gray) shaded area represents the parameter space where the consumers are better (worse) off in the equilibrium with free disposal than in the benchmark equilibrium, i.e., $W_S \geq W^*$ ($W_S > W^*$).

in period 1, the maximum per-unit profit determines whether the speculator's participation constraint is satisfied. According to equation (B.8), the per-unit profit is a function of the number of the strategic sellers in the market, m. Even though m is an exogenous parameter in our model, we are interested in how the competitiveness of the market affects speculation. In Figure 12, we plot the maximum per-unit profit, $\alpha p_{S2} - p_{S1}$, for a range of m when $a = 90$, $b = 1$, and $\bar{q}_S = 15$.

FIG. 12. The relationship between the maximum per-unit profit of the speculator and the number of strategic sellers. This figure plots the maximum per-unit profit of the speculator, $\alpha\, p_{S2} - p_{S1}$, for a function of the number of strategic sellers, $m \in [2, 20]$ in the marketplace – a measure of the ex-ante degree of competitiveness. We assume demand parameters, $a = 90$ and $b = 1$, and the storage capacity of the speculator, $\bar{q}_S = 15$.

The per unit profit of the speculator is a hump shape function of the number of the strategic sellers, m. It first increases with m, and reaches maximum when $m = 4$ and $m = 5$, and then decreases with m. Speculation can never occur in two extreme cases we do not consider in this numerical example. First, if there is a monopolistic seller in the market, the speculator cannot move the clearing price since the monopolist is fully aware of the strategy of the speculator. Second, if the market is fully competitive, the speculator cannot affect the price and make a profit. Thus, speculation can only happen in an oligopolistic market with not too many strategic sellers.

So far, in the economies we considered we assumed that there are no exogenous uncertainties. In the next section, we extend the analysis to an economy with demand shocks.[15]

III.B An Economy with Demand Uncertainty

We assume that the intercept term in the period 2 aggregate inverse demand function of consumers is given by: $a_2 = a + \epsilon \geq 0$, where ϵ is a zero-mean random variable and a is the intercept term in the period 1 aggregate inverse demand function. The value of ϵ is revealed to all agents at the beginning of period 2. As before, we assumed strategic sellers and the speculator are risk neutral. Further, we assume that, $\mathbb{P}(a_2 \geq b\,\bar{q}_S) = 1$, i.e., the lowest realized a_2 is above b times the speculator's maximum storage capacity. Since a_2 is known to all agents when they trade in period 2, the strategic sellers will adjust their supplies according to the realization of ϵ, and that will affect the market clearing price in period 2.

In the economy without the speculator, there is no connection between period 1 and period 2. The equilibrium outcomes in period 2 are given by:

$$q^* = \frac{a_2}{3\,b}, \quad p^* = \frac{a_2}{3}, \quad \text{and} \quad \pi^* = \frac{a_2^2}{9\,b}. \tag{III.1}$$

The equilibrium outcomes in period 1 will be the same as in the equilibrium without demand uncertainty and no speculator. Each strategic seller will earn a profit of $\frac{a^2}{9b}$ in period 1 and an expected profit of $\mathrm{E}\left[\pi^*\right]$ in period 2.

When there is a speculator and disposal is not available, the speculator will have to sell all his inventory in period 2 using a market order. Therefore, when disposal is not available, the market clearing price in period 2 is given by:

$$p_2^{*N} = \frac{a_2 - b\,q_{S1}}{3}. \tag{III.2}$$

[15] If we assume that the future demand is uncertain and the speculator is risk-averse, then the speculator needs a premium to undertake risky speculation. This will make profitable speculation less likely.

Each strategic seller's profit is given by:

$$\pi_2^{*N} = \frac{(a_2 - b\,q_{S1})^2}{9b}. \tag{III.3}$$

When free disposal is available, the speculator will optimally choose a combination of market and stop-loss orders as given below.Following the arguments in the proof of Lemma 2, it can be shown that the equilibrium market clearing price in period 2 is given by:

$$p_2^{*W} = \frac{2a_2 + b\,q_{S1}}{6}. \tag{III.4}$$

Suppose that the speculator submits his demand in the form of (II.9) in period 1, the strategic sellers will evaluate the benefit and cost of increasing supply to fill the speculator's limit order. The condition ensures the execution of the limit order:

$$\left(\frac{a - p_{S1}}{b} - \frac{a}{3b} + q_{S1}\right) p_{S1} + \mathrm{E}\left[\pi_2^{*N}\right] \geq \frac{a^2}{9b} + \mathrm{E}\left[\pi_2^{*}\right]. \tag{III.5}$$

In the above equation, the LHS is the expected profit of the strategic seller who unilaterally increases supply and the RHS is the expected profit if she supplies the same quantity as in the equilibrium of the economy with no disposal. It follows that the uncertainty of the period 2 demand does not change the equilibrium clearing price in period 1 and thus the optimal demand of the risk-neutral speculator remains unchanged. When the speculator is risk averse, the demand uncertainty deters speculation by making the speculator's profit risky.

III.C Equilibrium with Two Speculators

In this section, we examine the economy with two speculators, X and Y, and two strategic sellers, A and B, and show that the results in the earlier sections continue to hold with two speculators when the aggregate storage capacity is limited.

We first consider the case in which the speculators do not have access to disposal. Suppose speculators X and Y enter period 2 with inventories q_x and q_y. Each speculator maximizes his period 2 profit by taking the other speculator's strategy as given. The period 2 equilibrium supply and the market clearing price will be as given earlier in equation (II.6) and (II.7), with the aggregate inventory of the speculators replacing the inventory of the single speculator. Notice that the aggregate inventory will be $q_x + q_y$ or q_x or q_y or 0 depending on the period 1 clearing price and the prices and quantities specified in the two speculators' limit orders.

In period 1, speculators X and Y will choose prices p_x and p_y, and quantities q_x and q_y, for their respective limit orders. Similar to the economy with a single speculator, each speculator ensures that the strategic sellers will supply the amount they demand using limit orders. In addition, each speculator will take into account the limit order of the other speculator as well when submitting his limit order.

Let $\bar{q}_x$ and $\bar{q}_y$ denote the maximum storage capacity of the two speculators X and Y respectively; $\boldsymbol{p}_S(\cdot)$ denotes the period 1 best limit price choice for a single speculator defined in equation (II.15); and a and b are demand and sensitivity parameters as defined in equation (II.3). The following proposition summarizes an equilibrium in which the generated price spread with two speculators is the same as if there is a monopolistic speculator with the same aggregate storage capacity.

Proposition 3. *When the aggregate maximum storage capacity, $\bar{q}_x + \bar{q}_y \leq \frac{(5\sqrt{3}-6)a}{39\,b}$, the speculators X and Y will set the limit buy-quantity as $\bar{q}_x$ and $\bar{q}_y$ respectively and set the limit buy price as $\boldsymbol{p}_S(\bar{q}_x + \bar{q}_y)$. The price will be lower in period 1 when the speculators buy and higher in period 2 when the speculators sell. The price spread between period 1 and period 2 is maximized when the aggregate storage capacity is equal to $\frac{(5\sqrt{3}-6)a}{39\,b}$. The speculators will participate when the anticipated price spread is greater than the per unit storage cost.*

Proof: See Appendix A.

The results in proposition 3 show that the magnitude of the price spread between the two periods is independent of the number of speculators when the aggregate storage capacity is limited. When the aggregate capacity is relatively low, the two speculators will not compete with each other on the limit buy price. This is because each speculator's objective is to maximize the spread for a given limit buy-quantity after taking the rival's limit order as given. The price spread is an increasing function when the aggregate limit buy-quantity is less than $\frac{(5\sqrt{3}-6)a}{39\,b}$ which is the maximum per-unit cost that satisfies the speculator's participation constraint derived in Lemma 1. Moreover, each speculator will choose the maximum storage capacity available to him as the limit buy-quantity to maximize their own trading profits.

Proposition 3 does not imply that the equilibrium price spread is always independent of the number of speculators. Specifically, when the two speculators may compete on price, resulting in a smaller price spread, when their aggregate inventory capacity exceeds the threshold mentioned in proposition 3 but less than $\frac{9\,a}{(11+5\sqrt{61})\,b}$ which is the profit-maximizing inventory of the single speculator as shown in Section II.B.1. We do not provide detailed proof for the case when the aggregate inventory capacity is large but conjecture that the generated price spread in such a case will be equal to the speculators' storage cost.

Next, we consider the economy with disposal. In this economy, the speculators have more choice variables: period 1 limit price and quantity, and period 2 stop-loss price and quantity. To simplify the analysis, we limit the maximum quantity that the speculator can dispose of in period 2. Let $1-\underline{\alpha}$ denote the fraction of the acquired inventory that can dispose of without cost. We further assume that each speculator disposes of $1-\underline{\alpha}$ fraction or not to dispose of at all.

Let $\bar{q}_x$ and $\bar{q}_y$ denote the storage capacity of the two speculators X and Y respectively; $\boldsymbol{p}_S(\cdot)$ and $\boldsymbol{p}_{S2}(\alpha, q_{S1})$ respectively denote the period 1 best limit price and period 2 best stop-loss price chosen by the speculator in the economy with only one speculator, given by equation (II.15) and (A.6) in the appendix. We assume $b = 1$ for expositional convenience. Hence, the aggregate demand function becomes $p_t(Q_t) = a - Q_t$, for t, $t = 1, 2$.

The following proposition characterizes the equilibrium in which the equilibrium price spread with two speculators is the same as in the economy with only one speculator.

Proposition 4. *If the aggregate storage capacity, $\bar{q}_S = \bar{q}_x + \bar{q}_y$, is below $\frac{(5\sqrt{3}-6)a}{39}$ and the speculators have to choose between no disposal and disposing of a $(1 - \underline{\alpha})$ fraction of the acquired inventory, where $\underline{\alpha} \in \left[\frac{a-\bar{q}_S}{a+2\bar{q}_S}, 1\right]$, speculators X and Y will both supply $\underline{\alpha}$ fraction of their respective inventories using a market orders, and the remaining $1 - \underline{\alpha}$ fraction using stop-loss orders with the same stop-loss price $\boldsymbol{p}_{S2}\left(\underline{\alpha}, \bar{q}_S\right)$ in period 2, and limit buy-quantities $\bar{q}_x$ and $\bar{q}_y$ with the same limit buy price $\boldsymbol{p}_S(\bar{q}_x + \bar{q}_y)$ in period 1. Ceteris paribus, the equilibrium price spread in the with-disposal economy is greater than the equilibrium price spread in the without-disposal economy.*

Proof: See Appendix A.

Suppose that both speculators supply $\underline{\alpha}$ fraction of their inventories using market orders and the remaining $(1 - \underline{\alpha})$ fraction using stop-loss orders with a stop-loss price of $\boldsymbol{p}_{S2}\left(\underline{\alpha}, \bar{q}_S\right)$. In this case, each speculator makes a per-unit profit of $\underline{\alpha}\,\boldsymbol{p}_{S2}\left(\underline{\alpha}, \bar{q}_S\right)$ which is higher than the per-unit profit of $\boldsymbol{p}_{S2}\left(1, \bar{q}_S\right)$ when the speculators supply only using market orders in period 2.

Further, since the other speculator's stop-loss order will be triggered once one speculator chooses to supply the entire inventory using a market order, neither speculator will choose to give up the use of disposal technology. Moreover, the disposable fraction $1 - \underline{\alpha}$ cannot be too large that the speculator's per-unit profit after taking into account the disposed becomes lower than the per-unit profit in the no-disposal case, which leads to a lower bound of $\underline{\alpha}$.

IV Conclusion

We consider a two-period model economy with two strategic sellers and many atomistic consumers. In this model economy, there are no demand or supply uncertainties and the market clearing price is the same in both periods. We examine the consequences of a large speculator with access to the storage facility entering the market, specifically, the effect of speculative activities on the market-clearing prices and the welfare of all agents. The speculator has no independent supply. Hence the speculator buys in period 1 to supply in period 2. The speculator's strategies are known to all other agents. We show that the speculator lowers the market price while buying in period 1 and raises the market price while selling in period 2 through clever use of limit, market, and stop-loss orders. The speculator's use of limit buy order in period 1 changes the shapes of the aggregate demand curves in both periods that the strategic sellers face. That induces the strategic sellers to supply more in period 1 to meet the speculator's demand and reduce their supply to account for the dumping of the speculator's inventory in period 2, creating volatility that destabilizes the market in the sense that it reduces the profits of both strategic sellers. In period 2, if the speculator cannot dispose of any part of the inventory acquired in period 1 without selling, the speculator's activities make consumers better off in both periods. However, if the speculator can freely dispose of some of the inventory acquired in period 1 and sell only the rest in period 2, consumers are worse off in period 2, but better off overall when the two periods are taken together.

Our main results do not change if we introduce demand uncertainty when the speculator is risk-neutral, or introduce an additional speculator, but we do need additional restrictions on the storage capacity available to the speculators. We find that when there are multiple strategic sellers resulting in increased (but not perfect) competition among them, consumers can be made worse off overall as well. Their welfare gains in period 1 can be more than offset by their welfare losses in period 2. Our results suggest that destabilizing speculation that makes strategic suppliers and consumers worse off can occur even when all agents are rational and there is no asymmetric information among agents.

Appendix

A Proofs

Proof of Lemma 1.

To make the limit buy price satisfy the incentive compatibility constraints, one of the two strategic sellers has to find it profitable to supply more to meet the speculator's demand relative to the benchmark case without the speculator. Suppose that the speculator chooses the limit price as p_S for a given q_S and the two strategic sellers supply the benchmark quantity, q^*. If strategic seller A supplies more to meet the speculator's demand in full and reach the limit price, she has to supply $\frac{a-p_S}{b} - q^* + q_S$. This gives her a profit of $\left(\frac{a-p_S}{b} - q^* + q_S\right) p_S$ in period 1. If the strategic seller can have a higher profit when taking the two periods together, she will supply more to meet the speculator's demand, i.e.,

$$\text{(A.1)} \qquad \left(\frac{a-p_S}{b} - q^* + q_S\right) p_S + \frac{(a - b\, q_S)^2}{9b} \geq 2\pi^*,$$

where LHS is the unilateral deviating profit of the deviating strategic seller and RHS is her benchmark profit. The speculator chooses the lowest clearing price which satisfies condition (A.1) to minimize his inventory acquiring cost. When equation (A.1) holds as equality, we get the lowest limit price which is given by (II.15) in Lemma 1. This limit price makes the strategic sellers indifferent from supplying the benchmark quantity and supplying more to meet the speculator's demand.

To show the strategic sellers will not supply more than $\left(\frac{a-\boldsymbol{p}_S}{b} - q^* + q_S\right)$ when the speculator chooses a limit buy price p_S for a given q_S, we first note that any further increase in supply does not affect the strategic sellers' profit in period 2 when the speculator's limit order is fulfilled in period 1. The best response function of strategic seller A in period 1 is thus given by:

$$\text{(A.2)} \qquad \boldsymbol{q}_{A1}(q_{B1}) = \frac{a}{2b} - \frac{q_{B1}}{2} + \frac{q_S}{2}.$$

By substituting q_{B1} with q^* in the above equation, we get the optimal quantity for the deviating strategic seller to supply, however, this quantity is smaller than the unilateral deviating quantity $\left(\frac{a-p_S}{b} - q^* + q_S\right)$. So that the deviating strategic seller does not have an incentive to increase her supply further.

In addition, the strategic sellers will supply the speculator's demand in full rather than supplying only a fraction of the speculator's demand when the speculator's limit buy price is p_S and the demand is q_S. To see this, let's first consider that the speculator demands two different quantities $q_S^{(1)}$ and $q_S^{(2)}$, where $q_S^{(1)} < q_S^{(2)}$, and the speculator chooses two different limit prices $\boldsymbol{p}_S(q_S^{(1)})$ and $\boldsymbol{p}_S(q_S^{(2)})$ for the two quantities. Since the $\boldsymbol{p}_S(q_S)$ is a decreasing function of q_S, we have $\boldsymbol{p}_S(q_S^{(1)}) > \boldsymbol{p}_S(q_S^{(2)})$. Recall that the speculator chooses the limit price which makes the strategic sellers indifferent between supplying more and supplying the benchmark quantity. If the speculator chooses to demand $q_S^{(2)}$ at price $\boldsymbol{p}_S(q_S^{(2)})$, the deviating strategic seller will be worse off by only supplying $q_S^{(1)}$ of the speculator's demand. Thus, the deviating strategic seller will always fully supply the speculator's order. This completes the proof. □

Proof of Proposition 1.

According to Lemma 1, the speculator's limit price will be the clearing price in period 1 if the speculator demands q_S and chooses $\boldsymbol{p}_S(q_S)$ as the limit price. The period 2 price p_2, which is given by equation (II.7), depends on how many units of widgets the speculator acquired. Hence the spread between the period 2 and period 1 clearing prices is given by:

$$\boldsymbol{p}_2(q_S) - \boldsymbol{p}_S(q_S) = \frac{1}{6}\left(\sqrt{b\, q_S(4a + 13b\, q_S)} - 5b\, q_S\right) \tag{A.3}$$

The price spread reaches its maximum $\frac{(5-2\sqrt{3})a}{39} \approx 0.04a$ when $q_S = \frac{(5\sqrt{3}-6)a}{39b} \approx \frac{0.07a}{b}$. Since the speculator's participation constraint will be satisfied if his per unit profit $\boldsymbol{p}_2(q_S) - \boldsymbol{p}_S(q_S)$ is greater than the per unit storage cost c_s, the speculator will enter the market and affect the clearing prices in both periods when $c_S \leq \frac{(5-2\sqrt{3})a}{39}$ which completes the proof. □

Proof of Corollary 1.

When there is no storage cost, the speculator's profit is the product of the inventory and the price difference between period 1 and period 2,

$$q_S\big(\boldsymbol{p}_S(q_S) - \boldsymbol{p}_S(q_S)\big) \tag{A.4}$$

Given that speculator's order has to make the deviating strategic seller indifferent to supplying more, the aggregate loss comes from the strategic seller who does not deviate. Hence, the aggregate loss is given by:

$$2\pi^* - q^*\boldsymbol{p}_S(q_S) - \boldsymbol{\pi}_2(q_S) \tag{A.5}$$

where π^* and q^* are the single-period benchmark profit and supply of any strategic seller derived in (II.2), $\boldsymbol{\pi}_2(q_S)$ is the period 2 profit of any strategic seller when the speculator liquidates q_S with a market order derived in (II.8), and $\boldsymbol{p}_S(q_S)$ is the lowest limit price that the speculator can achieve in period 1 with a limit quantity of q_S in (II.15). With the conditions that $a > 0$, $b > 0$, and $q_S > 0$, we can verify that (A.5) is always greater than (A.4) which completes the proof. □

Proof of Lemma 2.

Entering period 2, the speculator has an inventory of q_{S1} units and chooses to use a combination of a stop-loss order and a market order to sell his inventory. He will maximize his revenue by optimally choosing the price and the quantity on his stop-loss order. For the stop-loss order not to be executed, one strategic seller has to reduce her supply to $\frac{a-p_{S2}}{b} - \frac{a-b\,q_{S1}}{3b} - \alpha q_{S1}$ so that the clearing price is equal to p_{S2} when the other strategic seller supplies $\frac{a-b\,q_{S1}}{3b}$ which is the period 2 supply in the equilibrium without disposal. The price-quantity combination of the speculator's stop-loss order has to guarantee that the strategic seller who unilaterally reduces supply satisfies the following condition:

$$\left(\frac{a-p_{S2}}{b} - \frac{a-b\,q_{S1}}{3b} - \alpha q_{S1}\right) p_{S2} = \frac{(a-b\,q_{S1})^2}{9b},$$

where on the LHS is the deviating strategic seller's period 2 profit when she reduces supply unilaterally and on the RHS is the period 2 profit in the equilibrium without disposal. This gives the maximum stop-loss price that satisfies the strategic sellers' incentive compatibility constraints for any given α,

$$\text{(A.6)} \qquad \boldsymbol{p}_{S2}(\alpha, q_{S1}) = p^* + \frac{b\,q_{S1}(1-3\alpha) + \sqrt{3b\,q_{S1}(1-\alpha)(4a - b\,q_{S1}(1+3\alpha))}}{6}.$$

Since the speculator's revenue can be written as $\alpha q_{S1} p_{S2}$ where p_{S2} is a function of α and q_{S1}, the speculator will choose an optimal α to maximize his revenue. This gives the optimal α as a function of q_{S1}: $\alpha = \frac{3a}{2a+b\,q_{S1}} - \frac{1}{2}$. Note that $q_{S1} \in [0, \frac{a}{b}]$ is a sufficient condition for $\alpha \in (0, 1]$. Substituting α into p_{S2} gives equation (II.22).

For p_{S2} to be the equilibrium price, we need to show that the strategic sellers do not have the incentive to increase supply when the clearing price is p_{S2}. To see this, note that any increase in supply will trigger the execution of the stop-loss order, i.e., the speculator sells all of his inventory which is what happens in the without disposal case. This completes the proof. □

Proof of Proposition 2.

Given that the clearing price in period 1 is given by (II.23) and the stop-loss price and the derived optimal α are given in Lemma 2, we can write the speculator's per unit profit, $\alpha p_{S2} - p_{S1}$, as

$$\frac{1}{6}\sqrt{b\,q_{S1}(4a + 13b\,q_{S1})} - \frac{7b\,q_{S1}}{12}.$$

Its derivative with respect to q_{S1} is always positive for $q_{S1} \in (0, \frac{a}{b}]$. Therefore, the speculator will always fully utilize his storage capacity, $\bar{q}_S$. To ensure that the speculator will participate in the game, his per unit storage cost cannot be higher than the maximum $\alpha p_{S2} - p_{S1}$ which occurs when $q_{S1} = \bar{q}_S$. This completes the proof. □

Proof of Corollary 3.

By substituting the limit buy price in (II.23) and the stop-loss price in Lemma 2 into the price spread $p_{S2} - p_{S1}$, it is easy to verify that its derivative is always positive when q_{S1} is between 0 and $\frac{a}{b}$. Thus, the price spread will increase as $\bar{q}_S$ increases. Similarly, substituting the p_{S1} and p_{S2} into the consumer surplus function, which is $w^* = \frac{1}{b}(a - p^*)^2$ for the benchmark equilibrium and $w^{*s} = \frac{1}{2b}(a - p_1)^2 + \frac{1}{2b}(a - p_2)^2$ for the equilibrium with free disposal, we can verify that the consumers are always better off when the two periods are taken together for any given limit quantity chosen by the speculator in period 1. This completes the proof. $\square$

Proof of Proposition 3.

Without loss of generality, we assume that $q_x < q_y$. Since $\boldsymbol{p}_S(\cdot)$ is a decreasing function, $\boldsymbol{p}_S(q_x+q_y) < \boldsymbol{p}_S(q_y) < P_S(q_x)$. We prove the proposition by first showing that both speculators will choose $\boldsymbol{p}_S(q_x+q_y)$ as their limit price when their aggregate storage capacity is below $\frac{(5\sqrt{3}-6)a}{39b} \approx \frac{0.07a}{b}$ and then showing that each of them will choose their maximum storage capacity as the limit buy-quantity.

The objective of speculator Y in period 1 is to maximize the trading profit. For a given quantity $q_y < \bar{q}_y$, the speculator Y's objective becomes choosing a limit buy price to maximize the price spread between the two periods subject to the incentive compatibility constraints. When the speculator X chooses $q_x < \bar{q}_x$ and $p_x = \boldsymbol{p}_S(q_x + q_y)$, the minimum limit buy price that satisfies the incentive compatibility constraints is $\boldsymbol{p}_S(q_x + q_y)$. If the speculator Y chooses any limit buy price $p_y < \boldsymbol{p}_S(q_x + q_y)$, the limit orders of the speculator X and Y will not be executed. If the speculator Y chooses any limit buy price $\boldsymbol{p}_S(q_x + q_y) \leq p_y < \boldsymbol{p}_S(q_y)$, the clearing price will be equal to $\boldsymbol{p}_S(q_x + q_y)$, which leads to a price spread of $\boldsymbol{p}_2(q_y) - \boldsymbol{p}_S(q_y)$. If the speculator Y chooses any limit buy price $p_y \geq \boldsymbol{p}_S(q_y)$, the clearing price will be equal to p_y since the deviating strategic seller is better off filling speculator Y's order in full. Moreover, the minimum period 1 price the speculator Y can get within the range $p_y \geq \boldsymbol{p}_S(q_y)$ is $\boldsymbol{p}_S(q_y)$, which leads to a price spread of $\boldsymbol{p}_2(q_y) - \boldsymbol{p}_S(q_y)$.

Note that the price spread, $\boldsymbol{p}_2(q_S) - \boldsymbol{p}_S(q_S)$, is an increasing function for $q_S \leq \frac{(5\sqrt{3}-6)a}{39b}$, therefore, the speculator Y is better off choosing the limit buy price $\boldsymbol{p}_S(q_x + q_y)$ relative to $\boldsymbol{p}_S(q_y)$. The argument also applies to speculator X. Hence, the period 1 clearing price is equal to $\boldsymbol{p}_S(q_x+q_y)$ when the speculator X and Y choose limit buy-quantity q_x and q_y respectively. Moreover, each speculator will choose their maximum storage capacity as the limit buy-quantity as each speculator's trading profit is an increasing function of their own limit buy-quantity. $\square$

Proof of Proposition 4.

In period 2, the speculators X and Y choose either to supply the entire inventory using market orders, which yield profits of $q_x\boldsymbol{p}_2(q_x + q_y)$ and $q_y\boldsymbol{p}_2(q_x + q_y)$ respectively, or supply $\underline{\alpha}$ fraction of their inventory using market orders and the rest $(1 - \underline{\alpha})q_x$ and $(1 - \underline{\alpha})q_x$ using stop-loss orders, with a certain stop-loss price.

When one speculator sets the stop-loss price at $\boldsymbol{p}_{S2}(\underline{\alpha}, q_x + q_y)$, derived in the proof of Lemma 2, the other speculator will set the same stop-loss price when $\underline{\alpha}\,\boldsymbol{p}_{S2}(\underline{\alpha}, q_S) \geq \boldsymbol{p}_2(q_S)$ which yields $\underline{\alpha} \in \left[\frac{a-q_S}{a+2q_S}, 1\right]$.

In period 1, proposition 3 shows that both speculators will demand their storage capacities, $\bar{q}_x$ and $\bar{q}_y$, using limit orders in period 1 when the maximum aggregate storage capacity is below $\frac{0.07a}{b}$. Together with the fact that $\underline{\alpha}\,\boldsymbol{p}_{S2}(\underline{\alpha}, q_S) - \boldsymbol{p}_S(q_S) \geq \boldsymbol{p}_2(q_S) - \boldsymbol{p}_S(q_S)$, we obtain the existence of the with-disposal equilibrium. $\square$

B Outcomes in the Oligopoly Equilibrium

We assume that the demand function is given in equation (II.1) and there are m identical strategic sellers. When there is no speculator in the market, the benchmark equilibrium outcomes are

$$q^* = \frac{a}{(m+1)\,b}, \quad \text{and} \quad p^* = \frac{a}{m+1}. \tag{B.1}$$

When a speculator with no access to disposal enters the market, the equilibrium clearing price and the supply of each strategic seller in period 2 is

$$q_2 = \frac{a - b\,q_{S1}}{(m+1)\,b}, \quad \text{and} \quad p_2 = \frac{a - b\,q_{S1}}{m+1}. \tag{B.2}$$

The speculator submits a limit order to acquire inventory in period 1, and the optimal limit buy price that the speculator chooses is

$$p_{S1} = p^* + \frac{1}{2}b\left(q_{S1} - \sqrt{\frac{4a(m-1)}{b(m+1)^2}q_{S1} + \frac{4}{(m+1)^2}q_{S1}^2 + q_{S1}^2}\right), \tag{B.3}$$

where $p^* = \frac{a}{m+1}$ is the benchmark clearing price. As the square root term is greater than q_{S1}, we have p_{S1} always lower than p^*. In equilibrium, p_{S1} is the clearing price in period 1.

When a speculator with access to free disposal enters the market, the speculator uses a combination of market order and stop-loss order to sell in period 2. The highest stop-loss he can choose for any given α and q_{S1} is

$$p_{S2} = \frac{b(m+1)\sqrt{\frac{(\alpha-1)q_{S1}(b\,q_{S1}(\alpha+(\alpha-1)m+3)-4a)}{b(m+1)}} + 2a - b\,q_{S1}(\alpha+(\alpha-1)m+1)}{2(m+1)}, \tag{B.4}$$

where q_{S1} is the inventory he acquired in period 1 and α is the fraction of inventory he sells using market order in period 2. To maximize his profit in period 2, the optimal α for the speculator is

$$\alpha = \frac{4a + b(m-3)q_{S1}}{4a + 2b(m-1)q_{S1}}. \tag{B.5}$$

Substituting the optimal α into equation (B.4), we have

$$p_{S2} = \frac{2a + b(m-1)q_{S1}}{2(m+1)}. \tag{B.6}$$

The derivative of p_{S2} with respect to m and q_{S1} are $\frac{b\,q_{S1}-a}{(m+1)^2}$ and $\frac{b(m-1)}{2(m+1)}$ respectively. In equilibrium, the clearing price in period 2 is equal to p_{S2}. Similar to the equilibrium without disposal, the speculator with access to free disposal submits a limit order in period 1 and chooses the same p_{S1} in equation (B.3) to acquire inventory. In equilibrium, the strategic sellers increase supply to meet the speculator's demand in period 1 and the clearing price is p_{S1}.

The price spread between the two periods in the equilibrium with free disposal is

$$p_{S2} - p_{S1} = \frac{\sqrt{b\,q_{S1}(4a(m-1) + b(m(m+2)+5)q_{S1})} - 2b\,q_{S1}}{2(m+1)}, \tag{B.7}$$

and the per unit profit, $\alpha p_{S2} - p_{S1}$, of the speculator when free disposal is available is given by

$$\alpha p_{S2} - p_{S1} = \frac{2\sqrt{b\,q_{S1}(4a(m-1) + b(m(m+2)+5)q_{S1})} - b(m+5)q_{S1}}{4(m+1)}. \tag{B.8}$$

References

Allen, F., Gale, D., 1992. Stock price manipulation. Review of Financial Studies 5, 503–529.

Attari, M., Banerjee, S., Noe, T. H., 2006. Crushed by a rational stampede: Strategic share dumping and shareholder insurrections. Journal of Financial Economics 79, 181–222.

Basak, S., Pavlova, A., 2016. A model of financialization of commodities. The Journal of Finance 71, 1511–1556.

Bianco, J., 1997. The mechanics of futures trading: Speculation and manipulation. Hofstra Law Review 6, 27.

Bohl, M., Sulewski, C., 2019. The impact of long-short speculators on the volatility of agricultural commodity futures prices. Journal of Commodity markets 16, 100085.

Boyd, N., Harris, J., Li, B., 2018. An update on speculation and financialization in commodity markets. Journal of Commodity Markets 10, 91–104.

Buraschi, A., Whelan, P., 2022. Speculation, sentiment, and interest rates. Management Science 68, 2308–2329.

Büyükşahin, B., Harris, J., 2016. Speculators, prices, and market volatility. Journal of Financial and Quantitative Analysis 51, 1545–1574.

Chari, V., Jagannathan, R., Jones, L., 1990. Price stability and futures trading in commodities. The Quarterly Journal of Economics 105, 527–534.

Chatterjea, A., Jarrow, R., 1998. Market manipulation, price bubbles, and a model of the u.s. treasury securities auction market. Journal of Financial and Quantitative Analysis 33, 255–289.

Chen, Z., Wang, C., 2022. Effects of intervention policies on speculation in housing market: Evidence from china. Journal of Management Science and Engineering 7, 233–242.

Cheng, I.-H., Xiong, W., 2014. Financialization of commodity markets. Annu. Rev. Financ. Econ. 6, 419–441.

Cooper, D., Donaldson, G., 1998. A strategic analysis of corners and squeezes. Journal of Financial and Quantitative Analysis 33, 117–137.

Cowing, C. B., 1965. Populists, plungers, and progressives: A social history of stock and commodity speculation, 1890-1936. Princeton University Press.

Dai, P., Xiao, D., Su, J., Ayub, B., 2022. Economic policy uncertainty and commodity market volatility: implications for economic recovery. Environmental Science and Pollution Research 29, 60662–60673.

De Long, J. B., Shleifer, A., Summers, L. H., Waldmann, R. J., 1990. Positive feedback investment strategies and destabilizing rational speculation. The Journal of Finance 45, 379–395.

Deaton, A., Laroque, G., 1996. Competitive storage and commodity price dynamics. Journal of Political Economy 104, 896–923.

Eichengreen, B., Rose, A., Wyplosz, C., 1995. Exchange market mayhem: the antecedents and aftermath of speculative attacks. Economic Policy 10, 249–312.

Friedman, M., 1953. Essays in positive economics. University of Chicago Press.

Gan, J., Tsoukalas, G., Netessine, S., 2021. Initial coin offerings, speculation, and asset tokenization. Management Science 67, 914–931.

Goldstein, I., Guembel, A., 2008. Manipulation and the allocational role of prices. The Review of Economic Studies 75, 133–164.

Hart, O., Kreps, D., 1986a. Price destablizing speculation. Journal of Political Economy XCIV, 927–952.

Hart, O. D., 1977. On the profitability of speculation. The Quarterly Journal of Economics pp. 579–597.

Hart, O. D., Kreps, D. M., 1986b. Price destabilizing speculation. Journal of Political Economy 94, 927–952.

Hertzberg, A., 2018. A theory of disclosure in speculative markets. Management Science 64, 5787–5806.

Hetzel, R., 2012. The great recession: market failure or policy failure? Cambridge University Press.

Kawai, M., 1983. Spot and futures prices of nonstorable commodities under rational expectations. The Quarterly Journal of Economics 98, 235–254.

Kim, A., 2015. Does futures speculation destabilize commodity markets? Journal of Futures Markets 35, 696–714.

Kliesen, K., 2021. The link between higher commodity prices and inflation. Economic Synopses 22, 3.

Kyle, A. S., Wang, A., 1997. Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test? The Journal of Finance LII, 2073–2090.

Liu, Y., Z., Z., Zhao, L., 2015. Speculation spillovers. Management Science 61, 649–664.

Lof, M., 2015. Rational speculators, contrarians, and excess volatility. Management Science 61, 889–1901.

Muth, J., 1961. Rational expectations and the theory of price movements. Econometrica 29, 315–335.

Newbery, D., 1984a. Commodity price stabilization in imperfect or cartelized markets. Econometrica 52, 563–578.

Newbery, D. M., 1984b. Commodity price stabilization in imperfect or cartelized markets. Econometrica pp. 563–578.

Newbery, D. M., 1987. When do futures destabilize spot prices? International Economic Review pp. 291–297.

Opler, T., Titman, S., 1994. Financial distress and corporate performance. The Journal of Finance 49, 1015–1040.

Peng, L., Röell, A., 2014. Managerial incentives and stock price manipulation. Managerial incentives and stock price manipulation. 69, 487–526.

Scheinkman, J. A., Xiong, W., 2003. Overconfidence and speculative bubbles. Journal of political Economy 111, 1183–1220.

Senate, 2014. Wall street bank involvement with physical commodities. Majority and Minority Staff Report, Permanent Subcommittee on Investigations, Committee on Homeland Security and Government Affairs, United States Senate.

Senator Cantwell, M., 2022. Floor speech. The United States Senate Discussion on The Transportation Fuel Market Transparency Act.

Stein, J. C., 1987. Informational externalities and welfare-reducing speculation. The Journal of Political Economy pp. 1123–1145.

Turnovsky, S. J., 1983. The determination of spot and futures prices with storable commodities. Econometrica pp. 1363–1387.

Włodarczyk, B., Szturo, M., 2019. Impact of Macroeconomic and Financial Factors on the Commodity Price Indexes in the Context of Financialization. In Contemporary Trends and Challenges in Finance, 4th Wroclaw International Conference in Finance.

Xiong, X., Duc Huynh, T., Wang, J., 2022. The impact of economic policy uncertainties on the volatility of european carbon market. Journal of Commodity Markets 26, 100208.

Xuanming, S., 2010. Optimal pricing with speculators and strategic consumers. Management Science 56, 25–40.