Leverage Dynamics and Learning about Economic Crises*

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Abstract

Models of learning about economic crises generate risk premia that rise at the onset of a crisis, but then fall as belief uncertainty fades. In contrast, empirical risk premia remain elevated during crises. We resolve this tension via leverage dynamics generated by the impact of learning on optimal default and capital structure decisions within a representative agent consumption-based model. Endogenously time-varying leverage creates a feedback loop: the learning-induced slow recovery in equity prices raises leverage, thereby further depressing equity values and keeping the equity premium and credit spreads persistently high as the crisis unfolds. We structurally estimate the model and show it closely matches the joint dynamics of consumption, equity risk premia, credit risk, and leverage, especially during crises, together with the term structure of credit risk and default probabilities.

JEL Classification: E32, E44, G12, G13, G32, G33

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1 Introduction

Equity returns, credit spreads, and corporate leverage fluctuate over time. Moreover, such fluctuations are of crucial interest to investors, who aim to make informed decisions related to capital allocation, investment, and borrowing. While financial economists have had success in understanding the average size of equity returns, credit spreads and leverage[1], understanding their joint fluctuations and connecting them to macroeconomic dynamics is more challenging.

During times of crisis, such as the Great Depression, cumulative equity returns fall for multiple years and take a long time to recover (see Ghaderi, Kilic, and Seo (2022)). Similarly, credit spreads rise together with corporate leverage (see Kuehn, Schreindorfer, and Schulz (2023)) but do not quickly return to pre-crisis levels. Existing models of asset prices, however, fail to replicate the dynamics of the equity market during the Great Depression, as we show in Figure 1. This is true for models of time-varying rare disasters (Gabaix, 2012; Wachter, 2013; Seo and Wachter, 2018) and even for models of learning (Wachter and Zhu, 2019) – such models successfully generate negative equity returns and an escalation in credit risk at the onset of a crisis but fail to replicate the long-lasting negative impact of crises as equity values recover too quickly relative to their empirical counterparts.

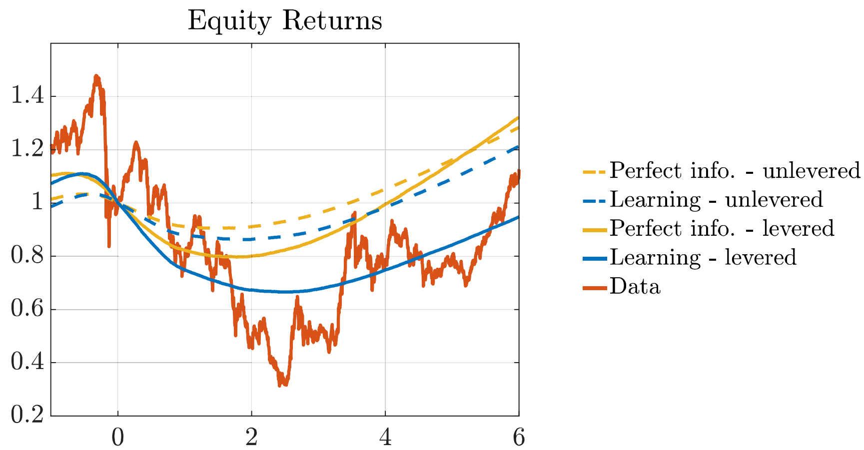
In this paper, we argue that leverage dynamics and learning about the stochastic frequency of jumps in consumption are the fundamental building blocks to understanding the long-lasting impact of crises on financial markets. To this end, we develop and estimate a model of asset prices with optimal default and leverage decisions to understand the joint time series of equity returns, credit spreads, and corporate leverage. We do so by embedding a firm-level model of optimal default and capital structure inside a consumption-based model with an Epstein-Zin representative agent with imperfect information about the consumption process's stochastic jump intensity but the ability to learn about its current value from observing consumption data.

Figure 1 depicts the cumulative drop in the U.S. equity market during the Great Depression (red line) and compares it with simulated data from a series of nested models. The full model features both learning and leverage – its implied equity dynamics match the data much more closely than any of the nested models. Learning alone (as in Ghaderi, Kilic, and Seo (2022); Wachter and Zhu (2019)) slows down the recovery in asset prices because agents learn quickly that they are in a high jump intensity regime but only discover slowly that they are not. While this feature is necessary, it is not sufficient to generate quantitatively realistic return dynamics during crises. With the addition of endogenously time-varying leverage, there is a feedback loop: the slow recovery in equity prices raises leverage, which

[1] See for example, Campbell and Cochrane (1999), Bansal and Yaron (2004) who focus on aggregate equity returns and Bhamra, Kuehn, and Strebulaev (2010b), Chen (2010) who study equity returns, credit spreads and leverage.

Figure 1: Great Depression – Equity Market

This figure presents cross-sectional averages of 10,000 model simulations, where the cumulative drop in annual consumption growth equals approximately 17%, akin to the decline observed during the Great Depression. The simulated data is aligned so that the beginning of a crisis corresponds to time 0, which is the beginning of 1930 in the data. The red line represents the daily, cum-dividend, inflation-adjusted cumulative equity return on the CRSP Index observed during the Great Depression. All other lines represent the daily, cum-dividend, cumulative equity return averaged across 125 individual firms and simulations generated by four different modes. Blue lines represent models with learning, yellow lines represent models with full information. Solid lines depict models with leverage, dashed lines depict models without leverage.



further reduces equity values. Thus, leverage amplifies the learning-induced slow recovery.

The statistical properties of consumption are an essential feature of all consumption-based models. We assume that consumption growth is driven by both Brownian shocks and jumps with stochastic intensity. We calibrate our consumption process to match the size and duration of the U.S. Great Depression and likelihood of consumption disasters from Barro and Ursua (2012) as well as the first four moments of U.S. consumption growth for the calm post-war sample. Confronting our model with the data in this manner forces us away from existing consumption disaster calibrations (see, for example, Rietz (1988) Martin (2008), Gabaix (2012)), and Wachter (2013)) and towards a calibration with starkly different properties. Relative to standard calibrations, our model features a low jump intensity state, representing normal times, and a high jump intensity state with more frequent jumps, representing severe recessions. Importantly, the typical high jump intensity episode does not lead to a disaster, as defined by Barro (2006). Economic disasters, where consumption drops by at least 10%, are rare and unfold slowly over time as a series of smaller jumps, as opposed to a single large decline. To acknowledge the challenges faced by economic agents in estimating a consumption process featuring downward jumps to represent crises, we make the stochastic consumption disaster intensity unobservable – the representative agent learns the intensity by observing consumption and engaging in Bayesian updating.

Given our calibration of the consumption process and a representative household equipped with Epstein-Zin preferences, we perform a structural estimation to determine parameter values that affect the firm-level earnings process and corporate financing decisions. The data on which we base our structural estimation span 2003 to 2022 and include CDX prices, firm-level returns, and leverage for the 125 firms in the CDX index at any time. Similar to Seo and Wachter (2018), we also price a CDX contract within a consumption-based asset pricing framework. As Seo and Wachter (2018) model default as the event whereby a firms value falls below an exogenous threshold, the CDX price is only a function of the aggregate jump intensity. In contrast, we model optimal default, where the distance-to-default is firm-specific. Consequently, the entire earnings distribution of the 125 index constituents is relevant for the pricing of CDX contracts. To make the estimation feasible, we approximate the earnings distribution with average cross-sectional leverage.

Given some predefined parameters for which the literature has strong priors, we estimate the size of idiosyncratic and aggregate Brownian risk, the idiosyncratic jump size, and bankruptcy cost parameters based on seven moments constructed from our sample: the mean and standard deviations of excess returns, leverage, and 5-year CDX rates together with the standard deviation of market excess return. Overall, the model closely matches the data well. The model generates a large equity premium of 9.5%, volatile equity returns of 33.8%, realistic leverage of 27.4%, smooth market returns with 14.4% dispersion, and a 5-year CDX rate of 72 basis points.

We also study the joint term structure of CDX rates and physical default probabilities. Our structural estimation was not designed to match physical default probabilities and targeted only the 5-year CDX rate. Nevertheless, our model can closely match the joint term structure of CDX rates and physical default probabilities from 1 year out to 10 years. At the short end, the average 1-year CDX rate is 21 basis points in the model compared to 22 basis points in the data; at the long end, the average 10-year CDX rate is 138 basis points in the model compared to 113 bps in the data. We stress that our model's ability to capture these two distinct term structures is not hardwired but instead provides an external validation of the underlying economic mechanisms.

Having estimated our model on data from 2003 to 2022, we simulate an event designed to represent the Great Depression and compare the resulting dynamics for the equity and credit market with the data. The major input for the simulation is the consumption path, which is designed to match the cumulative drop in annual consumption growth of approximately 17% during the Great Depression. The full model with learning and leverage replicates well the equity market losses during the Great Depression. Leverage magnifies losses to equity holders at the beginning of a crisis, and learning slows down the recovery during a crisis. In contrast, in the perfect information model, the equity market recovers swiftly when the economy switches into to low risk regime.

Similarly, learning magnifies credit risk. Even though firms are optimally more levered when they know the economic state, leverage and credit risk rise more in the full model with learning than in the perfect information model. The reason is a feedback loop between learning and leverage. The slow recovery in equity prices raises leverage, which further depresses equity values. Thus, leverage amplifies the learning-induced slow recovery.

Models of learning about crises must grapple with a significant challenge: in such models, (conditional) equity risk premia rise at the onset of a crisis but then swiftly decline. A key economic insight from Collin-Dufresne, Johannes, and Lochstoer (2016) exemplifies this problem: when the representative agent has a preference for earlier resolution of intertemporal risk, the equity risk premium increases with uncertainty about beliefs. Importantly, belief uncertainty only increases at the beginning of a crisis – as the crisis unfolds, it becomes increasingly obvious to the agent that the economy is not in a calm period, and so her belief uncertainty and the associated learning risk premium in the model declines, dragging down the model-based equity risk premium and credit risk measures, while their empirical counterparts continue to climb – Wachter and Zhu (2019) show this explicitly for the equity risk premium.[2] However, in the data, risk premia and measures of credit risk remain elevated during crises. Surprisingly, our model replicates this feature of the data: the levered equity risk premium and credit risk remain elevated as a crisis unfolds.

The impact of learning on optimal corporate financing decisions drives this pivotal result. As firm-level cashflows continue to decline, the distance-to-default shrinks, which increases leverage and, hence, the levered equity risk premium and credit risk. Crucially, the optimal default boundary does not depend on belief uncertainty but on the level of beliefs – a decrease in belief uncertainty does not disturb the leverage-based amplification mechanism. Instead, an increase in the agent's belief that the economy is in the high risk state raises the optimal default boundary, and so the impact of downward jumps in earnings on the distance-to-default is amplified by learning. Significantly, this amplification continues as an economic crisis builds, even though belief uncertainty declines. Therefore, the economic impact of learning on levered risk premia differs markedly from its unlevered equivalents – levered risk premia can rise even though belief uncertainty decreases, provided leverage increases.

[2] Wachter and Zhu (2019) study a model of rare crises with learning about the stochastic rate of crisis arrival – their Figure 1 shows clearly how the equity risk premium in the model shoots up when a disaster is realized and declines from thereon.

1.1 Related Literature

A growing literature seeks to provide a consumption-based explanation of not only stock market returns, but also credit risk and corporate leverage by integrating optimal default and capital structure decisions (see Leland (1994) and Goldstein, Ju, and Leland (2001)) within a representative-agent, consumption-based framework. Bhamra, Kuehn, and Strebulaev (2010b), Bhamra, Kuehn, and Strebulaev (2010a), and Chen (2010) assume the presence of long-run risk in consumption and do not attempt to replicate the observed times series dynamics. Kuehn, Schreindorfer, and Schulz (2023) assume consumption is subject to persistent crises and additionally analyze S&P100 option prices and the mean 5-year CDS rates for S&P500 firms. Relative to Kuehn, Schreindorfer, and Schulz (2023), we calibrate consumption to match the distribution of post-war consumption data, create additional time variation in asset prices and leverage via learning, and achieve an out-of-sample match of the joint term structure of physical default probabilities and CDX rates.

There is an extensive literature on learning and asset prices.[3] David (1997a) and Veronesi (2000) study how learning about unobservable expected consumption growth impacts equity returns. More recently, Collin-Dufresne, Johannes, and Lochstoer (2016) show that parameter learning creates long-run risk, which is priced when agents have a preference for earlier resolution of intertemporal risk. Johannes, Lochstoer, and Mou (2010) study how a combination of parameters, states, and model uncertainty in consumption-based models impacts asset return moments and the aggregate price-dividend ratio's sample path. The vast majority of this literature focuses on equity returns, including models with learning about rare disasters: Koulovatianos and Wieland (2016) and Wachter and Zhu (2019). Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015) study how learning impacts credit risk but does not feature optimal default decisions. While both Klein (2007) and Opp (2019) study optimal default impacted by learning, neither paper studies the interaction between optimal corporate financing decisions, equity returns, and credit risk. Hennessy and Radnaev (2016) develop a representative-agent model with a cross-section of firms subject to rare disasters, where the disaster arrival rate is stochastic and unobservable. While firm-level corporate financing decisions are optimal, firms only live for one period before being replaced. Consequently, each firm's leverage is chosen afresh each period, and so, aggregate leverage falls immediately when a disaster occurs. In our framework, firms live for relatively long time intervals, so leverage is not reset continuously, and therefore aggregate leverage rises throughout the majority of a crisis, and is a key driver of the resulting rise in the equity risk premium and CDX rates. In contrast with the existing literature, in our paper, learning impacts the intricate

[3] Ziegler (2003) and Pastor and Veronesi (2009) provide surveys of the literature.

interactions between levered asset prices and corporate financing decisions in a manner consistent with empirical observations.

Our paper is also related to the recent literature on CDX pricing. Seo and Wachter (2018) study CDX rates across tranches in a consumption-based model with time-varying rare events risk, where default occurs when firm value reaches or crosses an exogenous boundary. Collin-Dufresne, Junge, and Trolle (2022) model asset values under the risk-neutral measure and feature an exogenously time-varying default boundary with the aim of studying the extent to which equity option and CDX markets are integrated. Doshi, Ericsson, Fournier, and Seo (2021) also study the extent to which equity option and CDX markets are integrated, but model asset values under the physical measure. Unlike the above papers, we model cashflow risk to shareholders and bondholders separately, together with optimal capital structure and default decisions.

The literature on rare disasters employs a very different consumption calibration relative to our paper. The majority of the literature focuses exclusively on equity risk – see, for example, Rietz (1988), Barro (2006), Nakamura, Steinsson, Barro, and Ursua (2013), and Wachter (2013). Liu, Pan, and Wang (2005), Seo and Wachter (2019), and Barro and Liao (2021) extend the rare disaster approach to equity option prices. Gabaix (2012) studies a wide range of asset pricing puzzles with a time-varying rare disaster probability, but does not examine the role of optimal default and leverage. Christoffersen, Du, and Elkamhi (2017) use a consumption-based model with habit formation and constant disaster arrival rate to study credit risk, credit risk derivatives and equity derivatives, but does not consider optimal default and capital structure decisions or learning. In terms of the consumption calibration, Ghaderi, Kilic, and Seo (2022) also deviate from the usual disaster calibration and use a form of multilayered learning to match the behavior of the equity risk premium during crises, but they abstract from leverage and do not consider assets exposed to credit risk.

The remainder of the paper is organized as follows. Section 2 presents the asset-pricing block of the model: a representative agent consumption-based model with learning and a discussion of how consumption dynamics are calibrated. Section 3 presents the corporate finance block of the model: a firm-level model of optimal default and capital structure decisions is embedded within the asset pricing block, we derive levered equity prices, corporate bond prices, and describe how CDX rates are computed. Section 4 describes the data and presents the empirical results. Finally, Section 5 concludes. Appendix A summarizes notation, Appendix B includes omitted proofs, and Appendix C derives results for special case of no learning.

2 Consumption, Learning Dynamics, and the Stochastic Discount Factor

We embed a model of optimal corporate financing for a cross-section of firms inside a representative agent consumption-based asset pricing model. The key novelty of our model relative to the existing literature (see Bhamra, Kuehn, and Strebulaev (2010b) and Chen (2010)) is a common rare disaster shock affecting both aggregate consumption and firm-level earnings, where the disaster probability is stochastic and unobservable. The representative agent forms and updates subjective beliefs about the disaster probability by observing consumption. Crucially, the representative agent's beliefs impact her optimal capital structure and default decisions at the firm-level.

2.1 Consumption Dynamics

We describe the aggregate consumption process, which is superficially the same as Wachter and Zhu (2019). However, our approach to modeling rare disasters differs markedly from Wachter and Zhu (2019) and leads to a distinct calibration described in Section 2.2.

The exogenous aggregate consumption process is given by

$$\frac{dC_t}{C_{t-}} = \mu_c dt + \sigma_c dB_{c,t} + \left(e^{-Z_{c,t}} - 1\right) dN_t, \tag{1}$$

where B_c is a standard Brownian motion and N is a Poisson process with *unobservable* intensity λ_t under the objective physical probability measure $\mathbb{P}$. We interpret Brownian increments to consumption growth as small, highly frequent shocks, while the Poisson shock is less frequent but larger. When the Poisson process N increases by one, log consumption jumps by the random amount $-Z_c < 0$, where Z_c is an exponentially distributed random variable with mean $1/\epsilon_c > 0$ under $\mathbb{P}$.[4] The processes B_c, N, and Z_c are independent.

The unobservable jump intensity, λ_t, is driven by a continuous-time Markov chain which switches randomly between two risk-states: state L (the low risk state), where $\lambda_t = \lambda_L$ and state H (the high risk state), where $\lambda_t = \lambda_H$. The physical transition rate from state L into state H is ϕ_{LH}, while the physical transition rate from state H into state L is ϕ_{HL}.

The representative agent does not observe the physical transition intensity, λ_t, but she does know all other parameters for the consumption process, the physical transition intensities for the Markov chain and observes consumption. She therefore learns about the jump intensity and we assume she does

[4] We note that, conditional on a jump occuring the mean jump in consumption growth is given by

$$E_{t-}[e^{-Z_{c,t}} - 1 | dN_t = 1] = -\frac{1}{1+\epsilon_c} < 0,$$

and so we define $J_c = \frac{1}{1+\epsilon_c}$

so in a Bayesian manner, observing consumption and updating her prior belief that the jump intensity is in the high-risk state to form a posterior. We describe her preferences and learning dynamics in Section 2.3.

The traditional approach to modeling consumption disasters assumes such disasters unfold instantaneously in continuous-time models (see, e.g. Martin (2008) and Wachter (2013)), and over one time period in discrete-time models (see e.g. Rietz (1988), Gabaix (2012)). Empirically, disasters unfold over multiple periods, i.e. they are slow-moving and treating them as single period events may overstate their riskiness (see Constantinides (2008) and Julliard and Ghosh (2012)).

In contrast with the traditional approach, we assume disasters are slow-moving (see Nakamura, Steinsson, Barro, and Ursua (2013) and Ghaderi, Kilic, and Seo (2022)). Therefore, unlike the traditional approach, a single downward jump in consumption is *not* large enough to be a disaster. Instead, we define a consumption disaster as a path segment during which consecutive annual growth rates of consumption are negative and cumulatively result in a drop of more than 10% (Barro and Ursua (2012) use this definition). Therefore, several downward jumps in close succession are needed for a disaster to occur. We can achieve this within a continuous-time model with smaller jump sizes, but higher jump intensities relative to the traditional approach.

2.2 Calibration of Consumption Dynamics

In this section, we describe how we calibrate the exogenous aggregate consumption process defined in (1). The seven parameters we need to pin down are summarized in Panel A of Table 1 and the results of the calibration are given in Table 2.

We aim to match the dynamics of consumption growth for both (i) the 'long sample', going back to 1929 and thus containing the Great Depression and (ii) the calm 'post-war sample' starting in 1947. We do so by matching three statistical moments from the long sample and four from the post-war sample.

We now explain which features of the long-sample we calibrate to. Following Barro (2006), we define consumption disasters as consecutive negative annual consumption growth rates where the cumulative drop exceeds 10%. The data set of Barro and Ursua (2012) features 125 consumption disasters in 28 countries, dating back as far as 1870 and ending in 2009. In this data set, disasters have an average duration of 3.7 years (44 months), are associated with an average drop in total consumption expenditures of 21.6%, and a likelihood of occurring of 3.6%. The Great Depression in the United States is representative of these events, with a drop in total consumption expenditures of 20.8% over 44 months (August 1929 to March 1933).

A drawback of the international dataset of Barro and Ursua (2012) is that consumption growth measures total consumption expenditures, whereas asset pricing models are typically calibrated based on nondurables and services consumption because this measure corresponds more closely to the concept of consumption in the models. Since our aim is to match the joint dynamics of consumption growth for the long sample, containing the Great Depression, and the calm post-war sample, we use annual real per-capital log consumption growth of service and non-durable consumption expenditures from the BEA.

To mimic the long sample, we simulate 10,000 daily consumption paths for 141 years, which is the average sample length in Barro and Ursua (2012). We then time aggregate daily consumption to annual frequency and define disasters as in Barro (2006). For the long sample, we aim to match an average disaster size of 16.59% and an average disaster duration of 44 months, which are the respective values for the U.S. Great Depression. Since the long U.S. sample contains only one disaster, we take the likelihood of entering a disaster from Barro and Ursua (2012), which is close to the value in Wachter (2013).

In the post-war sample, the worst consumption drop occurred during the corona pandemic in 2020, when consumption growth fell by 5.11%. To mimic the post-war sample, we simulate 10,000 daily consumption paths for 74 years and only retain the consumption path when the worst cumulative annual consumption drop does not fall below 5.11%. For the post-war sample, we target the mean, standard deviation, skewness, and kurtosis of consumption growth.

Given these simulated consumption paths, we minimize the sum of squared relative deviations between model and data moments by choosing the consumption drift μ_c, consumption volatility σ_c, the jump intensities for the low and high state λ_L and λ_H, the transition rates between the two states ϕ_{LH} and ϕ_{HL}, and the average jump size ϵ_c.

Overall, the model matches all consumption moments very well. To fit the calm post-war sample, consumption growth has a drift $\mu_c = 0.024$, low volatility of $\sigma_c = 0.011$, a low jump intensity $\lambda_L = 0.17$, and an average duration of 8.2 years ($\phi_{LH} = 0.12$). These parameter values generate volatility of 1.5%, left skewness of -1.1, and kurtosis of 5.0 in annual consumption growth, similar to the post-war sample.

In contrast, the long sample features disasters like the Great Depression. The high jump intensity state requires 1.7 jumps per year on average ($\lambda_H = 1.7$), a short duration of 2.6 years ($\phi_{HL} = 0.38$), and an average jump size of $1/\epsilon_c = 0.025$. Importantly, not every high jump risk episode leads to a disaster, as defined by Barro (2006), because during the typical high jump risk episode, aggregate consumption drops by only 4.8%. Overall, disasters are rare and the parameter values generate disasters like the U.S. Great Depression with a likelihood of 3.6%.

In Figure 9, we compare the distribution of model-generated disasters with the distribution of international disasters collected by Barro and Ursua (2012) in terms of cumulative consumption drop and duration.

2.3 Learning Dynamics

Next, we describe the learning dynamics of the representative agent, her preferences, and the joint implications of her learning dynamics and preferences together with consumption dynamics for the stochastic discount factor. The representative agent can observe aggregate consumption, knows all fixed parameters governing consumption dynamics, but does *not* observe the physical arrival intensity of consumption jumps, i.e. λ_t is unobservable.

The representative agent uses her observations of aggregate consumption to update her subjective beliefs about the probability of being in the high-risk state. We define p_t to be the agent's posterior belief that the current state is H i.e. $\lambda_t = \lambda_H$

$$p_t = Pr(\lambda_t = \lambda_H | \mathcal{F}_t),$$

where $\mathcal{F}_t$ is the information set available to the agent up to time t. Her time-t subjective jump intensity for aggregate consumption is the belief-weighted arithmetic mean of jump intensities across states, i.e. $\tilde{\lambda}_t = \tilde{\lambda}(p_t) = p_t \lambda_H + (1 - p_t)\lambda_L$. If we assume that the representative agent is Bayesian, the dynamics of her subjective beliefs are given by the following stochastic differential equation:

$$dp_t = \kappa(f_H - p_{t-})dt + \sigma_p(p_{t-})\ (dN_t - \tilde{\lambda}(p_{t-})dt), \tag{2}$$

where $\kappa = \phi_{LH} + \phi_{HL}$ is the rate at which the Markov chain (which drives transitions in λ_t between λ_L and λ_H) converges to its long-run mean (under $\mathbb{P}$), $f_H = \phi_{LH}/\kappa$ is the long-run physical probability of the Markov chain being in the high-risk state, and $\sigma_p(p)$, given by

$$\sigma_p(p) = \frac{\lambda_H - \lambda_L}{\tilde{\lambda}(p)} p(1-p) \geq 0$$

is a measure belief uncertainty.[5]

The representative agent cannot observe the disaster intensity, but she knows that λ_t switches randomly between the values λ_H and λ_L. She learns about the current state of λ_t by observing realizations of N_t, i.e., jumps (or their absence) in consumption (see (1)).

Whenever there is a downward jump in consumption, i.e. $dN_t = 1$, the agent's belief the economy

[5] The long-run physical probability of the Markov chain being in the low-risk state is $f_L = 1 - f_H = \phi_{HL}/\kappa$.

is in the high-risk state jumps up by an amount equal to her belief uncertainty, i.e.

$$dp_t = \sigma_p(p_{t-}) \geq 0. \tag{3}$$

Hence, the upward revision in the agent's belief is largest when belief uncertainty is maximized, i.e. when $p_{t-} = \sqrt{\lambda_L}/(\sqrt{\lambda_L} + \sqrt{\lambda_H})$.[6]

Between jumps, the agent's belief that the economy is in the high-risk state either grows logistically or decays exponentially towards a long-run belief p^*, given in (B3). If the agent's initial belief is above its long-run value, i.e. $p_0 > p^*$, belief dynamics will consist of exponential decay towards p^*, punctuated by upward jumps whenever there is a downward jump in consumption, as summarized in Figure 2. The size of the downward jump in consumption is unrelated to jump intensity and therefore has no impact on the agent's belief she is in the high-risk state. However, a slowly unfolding disaster, which is a series of downward jumps, will lead to a series of upward jumps in the agent's belief she is in the high-risk state, with little time for exponential decay to work, creating a substantial increase in risk perceptions.[7]

2.4 Preferences and the Stochastic Discount Factor

The representative agent has the continuous-time analog of Epstein-Zin-Weil preferences with an elasticity of intertemporal substitution, ψ, which equals 1. Thus, the representative agent's value function is given by

$$J_t = \tilde{E}_t \int_t^\infty f\left(C_s, J_s\right) ds, \tag{4}$$

where f is given by the normalized Kreps-Porteus aggregator:

$$f(c, v) = \beta(1-\gamma)v \ln\left(c/h^{-1}(v)\right), \tag{5}$$

for

$$h\left(x\right) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma \geq 0, \gamma \neq 1, \\ \ln x, & \gamma = 1, \end{cases}$$

[6] It is also useful to define the jump in the belief as

$$J_p(p) = \frac{\lambda_H - \lambda_L}{\tilde{\lambda}(p)}(1-p) = \frac{\sigma_p(p)}{p}.$$

[7] We shall denote the probability measure representing the agent's subjective beliefs via $\tilde{\mathbb{P}}$, which we define as follows. Let A be an event realized at time $T > t$ and let I_A be the indicator function associated with A. Now, $\tilde{E}_t[\cdot]$ denotes the time-t conditional expectation operator under $\tilde{\mathbb{P}}$ and $\tilde{E}_t[I_A]$ is the time-t conditional probability of event A, which we define via $\tilde{E}_t[I_A] = E_t\left[\frac{\tilde{M}_T}{\tilde{M}_t} I_A\right]$, where $\tilde{M}$ is the following exponential martingale under $\mathbb{P}$: $\frac{d\tilde{M}_t}{\tilde{M}_{t-}} = \left(\frac{\tilde{\lambda}_{t-}}{\lambda_{t-}} - 1\right)(dN_t - \lambda_{t-}dt)$, $\tilde{M}_0 = 1$.

where β is the rate of time preference and γ is the coefficient of relative risk aversion.[8]

The representative agent's value function is of the form

$$J(C_t, p_t) = h(e^{V(p_t)} C_t), \tag{6}$$

where the function $V(p_t)$ (see Figure 3) captures how learning about the (physical) intensity of jumps in consumption impacts the agent's utility.[9]

We now explain how learning dynamics amplify the welfare losses stemming from downward jumps in consumption. When consumption jumps downwards, this leads to a direct downward jump in welfare driven by C, as we can see from (6). However, the agent's belief that the economy is in the high-risk state also jumps upwards by an amount equal to the level of belief uncertainty, causing $V(p_t)$ to jump downwards (Figure 3 shows $V(p_t)$ is monotonically decreasing in p_t), creating an additional downward jump in welfare.

We can see from (3) and (6) that size of the learning based amplification of downward jumps in consumption depends on belief uncertainty, and is equivalent to an additional jump in log consumption, given by $a(p_{t-})$, where

$$a(p_{t-}) = V\left(p_{t-} + \sigma_p(p_{t-})\right) - V(p_{t-}) < 0 \tag{7}$$

and p_{t-} is the agent's belief that she is in the high-risk state just before the consumption jump and $p_{t-} + \sigma_p(p_{t-})$ is her updated belief just after. For example, from Figure 4, we can see that when the agent believes the economy is in the high-risk state with probability 0.2, the additional impact of a downward jump on welfare via the learning channel is equivalent to a downward jump in consumption of approximately 5%. Naturally, learning-based amplification attenuates as the agent becomes more certain of the economy's state because she perceives she has less to learn.

The agent's preference for early resolution of intertemporal consumption risk means that the learning-based amplification of utility losses from downward consumption jumps impacts both the price of consumption across time via the demand for precautionary savings and the price of consumption across states, which is encoded by the stochastic discount factor, π.

Beliefs are risky and because the agent has a prefers to resolve intertemporal risk more speedily, belief uncertainty is priced in addition to jump-risk. This is powerful, because learning amplifies the already substantial jump-risk price.

[8]The continuous-time version of the recursive preferences introduced by Epstein and Zin (1989) and Weil (1990) is known as stochastic differential utility (SDU), and is derived in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness. Kraft and Seifried (2010) show the version of SDU we use is well defined under a mixed Brownian-Poisson filtration.

[9]Proposition B1 in Appendix B shows that $V(p)$ satisfies a functional differential equation with a boundary condition at $p = p^*$.

The learning-driven amplification of welfare losses from downward consumption jumps will increase the value of a consumption unit in a state where consumption jumps downward relative to a state where there is no jump. To see this more explicitly, we consider the representative agent's equilibrium stochastic discount factor (SDF), denoted by π.

Proposition 1 *The dynamics of the equilibrium stochastic discount factor π_t are given by*

$$\frac{d\pi_t}{\pi_{t-}} = -r(p_{t-})dt - \Theta_B dB_{c,t} + [\Theta_J(Z_{c,t}) + \Theta_a(p_{t-}, Z_{c,t})]dN_t - (\lambda^{\mathbb{Q}}(p_{t-}) - \tilde{\lambda}(p_{t-}))dt, \tag{8}$$

where the locally risk-free rate is given by

$$r(p_t) = \beta + \mu_c - \gamma\sigma_c^2 - \tilde{\lambda}^{\mathbb{Q}}(p_t)\frac{J_c}{1 - J_c\gamma},$$

and the price of Brownian risks in log consumption is given by

$$\Theta_B = \gamma\sigma_c. \tag{9}$$

$\Theta_J(Z_{c,t})$ is the price of static jump risk, given by

$$\Theta_J(Z_{c,t}) = e^{\gamma Z_{c,t}} - 1 \approx \gamma Z_{c,t}$$

and $\Theta_a(p_{t-}, Z_{c,t})$ is the dynamic learning risk price

$$\Theta_a(p_{t-}, Z_{c,t}) = \Theta_J(Z_{c,t})[e^{-(\gamma-1)a(p_{t-})} - 1] \approx -(\gamma - 1)a(p_{t-})\Theta_J(Z_{c,t}) \tag{10}$$

The risk-neutral intensity rate for jump arrivals $\lambda^{\mathbb{Q}}(p_{t-})$ is related to the corresponding subjective intensity $\tilde{\lambda}(p_{t-})$ via

$$\lambda^{\mathbb{Q}}(p_{t-}) = \omega(p_{t-})\tilde{\lambda}(p_{t-}), \tag{11}$$

where $\omega(p_{t-})$ is a risk distortion factor, given by

$$\omega(p_{t-}) = \frac{1 - J_c}{1 - J_c(1+\gamma)} + \frac{1 - J_c}{1 - J_c(1+\gamma)}\left[e^{-(\gamma-1)a(p_{t-})} - 1\right]. \tag{12}$$

Without downward jumps in consumption, the risk-free rate is given by the standard expression $\beta + \mu_c - \gamma\sigma_c^2$. The only aggregate consumption risk is then standard Brownian risk, which carries the price Θ_B given by the standard expression (9) – we know from Mehra and Prescott (1985) that the risk price Θ_B can only create an empirically realistic equity premium magnitude if relative risk aversion γ is very large, because consumption growth volatility σ_c is relatively low – 1.13% per annum in our calibration.

The risk of a downward jump in consumption creates additional demand for precautionary savings, which depresses the risk-free rate by the amount $\tilde{\lambda}^{\mathbb{Q}}(p_t)\frac{J_c}{1-J_c\gamma}$ (provided $J_c < 1/\gamma$). This additional demand for precautionary savings has two components: the desire to save in the face of static disaster risk, which contributes $\frac{1-J_c}{1-J_c(1+\gamma)}$ to the risk distortion factor and a desire to save in the face of learning induced dynamic risk, which contributes $\frac{1-J_c}{1-J_c(1+\gamma)}\left[e^{-(\gamma-1)a(p_{t-})}-1\right]$ to the risk distortion factor, as shown in (12). We can see from Figure 6 that as the agent's belief she is in the high-risk state rises, so does her demand for safe assets, driving up the risk-free bond price and depressing the equilibrium risk-free rate.

The upwards jump in the SDF, which occurs when there is a downward jump in consumption, is the price of jump risk and has two components. The first component is given by $\Theta_J(Z_{c,t})$, would be present in a static disaster risk model, is independent of the agent's beliefs, and is driven by the size of the log consumption jump $Z_{c,t}$ together with the agent's relative risk aversion γ. This component drives the equity premium in Rietz (1988) and Barro (2006). The second component, given by $\Theta_a(p_{t-}, Z_{c,t})$, is present because learning impacts the agent's utility. As we can see from (10), $\Theta_a(p_{t-}, Z_{c,t})$ depends on the size of the learning-based amplification in utility losses from the downward jump, $a(p_{t-}) < 0$, and the extent to which the agent prefers the earlier resolution of intertemporal risk, $\gamma - 1/\psi = \gamma - 1 > 0$:

$$\frac{\pi_t}{\pi_{t-}} - 1 = \overbrace{\underbrace{e^{\gamma Z_{c,t}} - 1}_{=\Theta_J(Z_{c,t})>0}}^{\substack{\text{static disaster risk} \\ \text{(power utility)}}} + \overbrace{\underbrace{e^{\gamma Z_{c,t}}\left[e^{-(\gamma-1)a(p_{t-})} - 1\right]}_{=\Theta_a(p_{t-},Z_{c,t})>0}}^{\substack{\text{dynamic learning risk} \\ \text{(EZW preferences, } \gamma>1)}}.$$

The learning-induced component of the jump-risk price is also time-varying, so in addition to increasing the equity premium's magnitude, it introduces endogenous time variation, as in Wachter and Zhu (2019).

The risk distortion factor $\omega(p_{t-})$ (see Figure 5) summarizes how jump risk is priced into the subjective consumption jump arrival intensity, and creates a wedge between risk-neutral and subjective consumption jump arrival intensities. We can decompose the risk distortion factor into two parts. The first part is the physical mean of the static disaster risk jump in the SDF, i.e., $E_{t-}[e^{\gamma Z_{c,t}}|dN_t = 1] = \frac{1-J_c}{1-J_c(1+\gamma)}$ and the second part is driven by dynamic learning risk, i.e., $e^{-(\gamma-1)a(p_{t-})}$. Therefore, the risk-distortion factor is a function of beliefs and is increasing with belief uncertainty, inheriting its shape, as shown in Figure 5. The risk-neutral jump arrival intensity is $\lambda^{\mathbb{Q}}(p_t)$ at time-t, given by (11).

3 Firms' Dynamics and CDX Prices

In this section, we embed a firm-level structural model of optimal default and capital structure inside the consumption-based model described in Section 2. We use our model to price firm-level defaultable debt and hence construct a price for CDX, an index of credit default swaps on investment-grade firms.

Our approach differs from Seo and Wachter (2018), Doshi, Ericsson, Fournier, and Seo (2021), and Collin-Dufresne, Junge, and Trolle (2022). In our model CDX prices depend directly on corporate bond prices derived from a structural model where firm-level earnings are exogenous, there is a stochastic discount factor from a consumption-based model, and default and capital structure decisions are made optimally, as in Bhamra, Kuehn, and Strebulaev (2010b) and Chen (2010). In contrast, Seo and Wachter (2018) CDX prices depend on an asset value process, as in Black and Cox (1976), where default occurs when asset values fall below an exogenous and constant barrier. In Collin-Dufresne, Junge, and Trolle (2022), firm values dynamics are specified exogenously under the risk-neutral probability measure, and while the firm-value default boundary is time-varying, it remains exogenous.

3.1 CDX Pricing

In this section, we explain how the price of a CDX depend on the prices of the underlying corporate bonds and firm-level default decisions. In subsequent sections, we explain the framework we use to derive corporate bond prices and endogenous default decisions.

We shall price Markit's North American Investment Grade CDX Index (the CDX.NA.IG Index commonly known as the "IG Index"), which is composed of 125 of the most liquid North American firms with investment grade credit ratings that trade in the CDS market. The index composition is chosen twice annually at times known as roll dates, which we denote by t_{roll}.[10] At each roll date, the CDX index is issued with six maturities: 1 year, 2 years, 3 years, 5 years, 7 years, and 10 years.

We now describe how the CDX index value is related to the corporate bond prices of the firms in the index. Let the set of firms in the index defined at the roll date t_{roll} be $\mathcal{N}_F(t_{\mathrm{roll}})$ and the number of firms in the index be $N_F(t_{\mathrm{roll}})$.

Let the time-t nominal price of the corporate debt issued by firm k be $D^{\$}_{k,t}$. The random default time of firm k's debt is denoted by $\tau_{D,k}$ and so we can define the nominal stochastic recovery rate

$$R^{\$}_{\tau_{D,k}} = \frac{D^{\$}_{k,\tau_{D,k}}}{D^{\$}_{k,t_{\mathrm{issue}}}},$$

which is the ratio of corporate debt value for firm k at default relative to the value at time of issuance, t_{issue} We use $n_{t,s}(t_{\mathrm{roll}})$ to represent the fraction of firms in the index defined at the roll date t_{roll}, that have defaulted between the times t and $s > t$:

$$n_{t,s}(t_{\mathrm{roll}}) = \frac{1}{N_F(t_{\mathrm{roll}})} \sum_{k \in \mathcal{N}_F(t_{\mathrm{roll}})} 1_{\{t < \tau_{D,k} \leq s\}}.$$

[10]The two roll dates for a given year are September 20 (or the Business Day immediately thereafter in the event that September 20 is not a Business Day) and March 20 (or the Business Day immediately thereafter in the event that March 20 is not a Business Day).

The key state variables which impact the value of the CDX index are $\{R^{\$}_{\tau_{D,k}}\}_{k\in\mathcal{N}_F(t_{\text{roll}})}$ and the stochastic process $n_{t,s}(t_{\text{roll}})$.

The $N_F(t_{\text{roll}})$ firms in the index are selected according to a pre-defined and publicly known set of rules (e.g. credit quality). A CDX contract issued at the roll date t_{roll} has a finite time to maturity, denoted by $T-t$, where t is the current time ($t \in [t_{\text{roll}}, T)$) and $T \in \{t_{\text{roll}}+1, t_{\text{roll}}+2, t_{\text{roll}}+3, t_{\text{roll}}+5, t_{\text{roll}}+7, t_{\text{roll}}+10\}$.

A CDX contract is a swap between a protection buyer and a protection seller. The protection buyer receives cashflows from the protection seller which are decreasing in the value of the corporate bonds in the index. The protection seller receives cashflows from the protection buyer which are fixed in nominal terms and are payments for the insurance the protection seller provides to the protection buyer.

We first describe the cashflows received by the protection buyer. If firm k in the index defaults at time $\tau_{D,k}$, the protection buyer receives a cashflow of $\$1 \cdot \frac{1}{N_F(t_{\text{roll}})} \cdot (1 - R_{\tau_{D,k}})$, where $R^{\$}_{\tau_{D,k}}$ is the recovery rate for firm k's debt. We can see that if the recovery rate is zero, then the cashflow is $\$1 \cdot \frac{1}{N_F(t_{\text{roll}})}$. Therefore, the sum of the nominal payoffs received by the protection buyer up until time $s > t$ is

$$L^{\$}_{t,s}(t_{\text{roll}}) = \$1 \cdot \frac{1}{N_F(t_{\text{roll}})} \sum_{k\in\mathcal{N}_F(t_{\text{roll}})} 1_{\{t<\tau_{D,k}\le s\}} \left(1 - R^{\$}_{\tau_{D,k}}\right),$$

which is increasing in the losses an investor in the bonds within the index would have made and is known as the cumulative loss. The nominal present value of the cumulative loss is given by

$$\text{Prot}^{\$}(T-t) = \$1 \cdot \frac{1}{N_F} \cdot \sum_{k\in\mathcal{N}_F} \tilde{E}_t \left[\int_t^T \frac{\pi^{\$}_{\tau_{D,k}}}{\pi^{\$}_t}(1 - R^{\$}_{\tau_{D,k}}) 1_{\{t<\tau_{D,k}\le s\}} ds\right],$$

where, for ease of notation, we have suppressed the dependence on the roll date. Using increments in L_s, we can write the nominal present value of protection payoffs compactly as

$$\text{Prot}^{\$}(T-t) = \tilde{E}_t \left[\int_t^T \frac{\pi^{\$}_s}{\pi^{\$}_t} dL^{\$}_{t,s}\right].$$

We now describe the cashflows received by the protection seller. The protection seller receives insurance payments which amount to $\$1 \cdot (S_{\text{CDX}} \cdot \frac{1}{4}) \cdot (1 - n_{t,s})$ every quarter.[11] In addition, as payments are made in arrears, the protection seller receives an accrued premium as compensation for the time the defaulted entity was covered since the last scheduled payment. Suppose, for example, firm k defaults between dates $t + \frac{1}{4}$ and t, then firm k was covered for an additional timespan of $(\tau_{D,k} - t)$ since the last quarterly payment: the accrued premium amounts to $\$1 \cdot (S_{\text{CDX}} \cdot \frac{1}{4}) \cdot 4(\tau_{D,k} - t) \cdot dn_{t,\tau_{D,k}}$, where $dn_{t,\tau_{D,k}} = \frac{1}{N_F}$ as we assumed a singular default. Aggregating both the expected discounted

[11] It is market convention to quote quarterly paid spreads in annual terms, hence $S_{\text{CDX}} \cdot \frac{1}{4}$.

scheduled and default triggered payments gives the following expression for the nominal present value of cashflows received by the protection seller

$$\begin{aligned}\text{Prem}\left(T-t,S_{\text{CDX}}\right)=&\$1\cdot S_{\text{CDX}}\cdot\frac{1}{4}\cdot\tilde{E}_t\left[\sum_{m=1}^{4T}\int_t^{t+\frac{1}{4}m}\frac{\pi_s^\$}{\pi_t^\$}ds\left(1-n_{t,t+\frac{1}{4}m}\right)\right.\\&\left.+\int_{t+\frac{1}{4}(m-1)}^{t+\frac{1}{4}m}\int_t^s\frac{\pi_u^\$}{\pi_t^\$}du\,4\left(s-t-\frac{1}{4}(m-1)\right)dn_{t,s}\right].\end{aligned}$$

Finally, S_{CDX} is set such that present values of protection and premium payments are equal when the contract commences at time t_{roll}, and so

$$S_{\text{CDX}}=\frac{\text{Prot}\left(T-t_{\text{roll}}\right)}{\text{Prem}\left(T-t_{\text{roll}},\$1\right)}.$$

The time-t value of a CDX contract on a notional value of \$1 from the perspective of the protection buyer is given by

$$\begin{aligned}&\text{Prot}^\$\left(T-t\right)-S_{\text{CDX}}\cdot\text{Prem}\left(T-t,\$1\right)\\&=\tilde{E}_t\left[\int_t^T\frac{\pi_s^\$}{\pi_t^\$}dL_{t,s}^\$\right]\\&-S_{\text{CDX}}\cdot\frac{1}{4}\cdot\tilde{E}_t\left[\sum_{m=1}^{4T}\int_t^{t+\frac{1}{4}m}\frac{\pi_s^\$}{\pi_t^\$}ds\left(1-n_{t,t+\frac{1}{4}m}\right)+\int_{t+\frac{1}{4}(m-1)}^{t+\frac{1}{4}m}\int_t^s\frac{\pi_u^\$}{\pi_t^\$}du\,4\left(s-t-\frac{1}{4}(m-1)\right)dn_{t,s}\right].\end{aligned}$$

We now describe the model of optimal default and corporate financing, which we embed inside the consumption-based model of Sections 2–2.3.

3.2 Firms' Earnings Dynamics

There are K firms in the economy. The real earnings process for firm $k\in\{1,\ldots,K\}$ is given by $X_{k,t}$ with

$$\frac{dX_{k,t}}{X_{k,t-}}=\mu_x dt+\sigma_x^{\text{id}}dB_{x,k,t}+\sigma_x^{\text{sys}}dB_{x,t}+(e^{-Z_{k,t}}-1)dN_t-dN_{k,t},$$

where μ_x is the expected earnings growth rate, $B_{x,t}$ and $B_{x,k}$ are standard Brownian motions under $\mathbb{P}$, where $dB_{x,t}$ represents the systematic shock the earnings of firm k while $dB_{x,k,t}$ represents an idiosyncratic shock the earnings of firm k. We assume that $E_t[dB_{c,t}dB_{x,t}]=\rho_{cx}dt$ and $E_t[dB_{c,t}dB_{x,k,t}]=E_t[dB_{x,k,t}dB_{x,k',t}]=0$ for $k'\neq k$. Therefore, σ_x^{id} is the idiosyncratic volatility from Brownian shocks, σ_x^{sys} is the systematic volatility from Brownian shocks, and ρ_{cx} is the correlation between the Brownian shock to consumption and systematic Brownian shock to earnings.

Firm k's earnings are subject to aggregate jump risk via the same Poisson shock, dN_t, which impacts aggregate consumption. However, the impact of this aggregate shock is heterogeneous across firms. The size of the jump in the logarithm of firm k's earnings, is given by $Z_{k,t}$, where $Z_{k,t}$ is an exponentially distributed random variable independent across firms and also independent of $Z_{c,t}$ with common mean $1/\epsilon_x = \varphi/\epsilon_c > 0$.[12]

Distance-to-default will be endogenously heterogeneous across firms because of heterogeneity in jump sizes and idiosyncratic Brownian shocks. Hence, a downward jump in consumption will lead some firms to default and change the cross-sectional distribution of distance-to-default for firms that do not default.

In addition, we assume that an individual firm can exogenously exit the economy when the idiosyncratic Poisson shock $dN_{k,t}$ equals 1 (which drives earnings to zero), which occurs with exogenous intensity λ_x under $\mathbb{P}$.[13] This type of shock can be interpreted economically as a particular firm's earnings technology becoming unprofitable due to, e.g. a firm-specific event such as prohibitive regulation, obsolete products, or the departure of key employees.

Observing the panel of firms' earnings does not reveal more information about the consumption jump arrival intensity because each firm's earnings is hit by the same jump shock as consumption. Therefore, the learning dynamics described in Section 2.3 still apply.

3.3 Learning, Optimal Default, Capital Structure, and Asset Prices

We now characterize the two-way feedback between prices of a firm's levered equity and its defaultable corporate debt and optimal default and capital structure decisions. The novelty of our model lies in its ability to shed light on how slowly unfolding disasters and beliefs about their arrival rate influence this feedback.

We follow the standard earnings-based approach of modelling corporate debt and optimal capital structure decisions as seen in e.g. Goldstein, Ju, and Leland (2001), Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010b), Bhamra, Kuehn, and Strebulaev (2010a), and Chen (2010). We assume initial firm owners of an all-equity firm issue a claim that entitles its owner to demand a constant flow of payments with coupon rate c for as long as the firm is solvent. Subtracting these interest payments from the aggregate earnings $X_{k,t}$ yields the payoff to the owners of levered

[12] We note that, conditional on a jump occuring the mean jump in a firm' s earnings growth is given by

$$E_{t-}[e^{-Z_{k,t}} - 1|dN_t = 1] = \tilde{E}_{t-}[e^{-Z_{k,t}} - 1|dN_t = 1] = -\frac{1}{1+\epsilon_x} < 0,$$

and so we define $J_x = \frac{1}{1+\epsilon_x}$. Furthermore, in general wtih $q(\cdot,\cdot) : \mathbb{R}^2 \to \mathbb{R}$, $\tilde{E}_{t-}[q(Z_{c,t}, Z_{x,t})|dN_t = 1] = E_{t-}[q(Z_{c,t}, Z_{x,t})|dN_t = 1]$, because we condition on the jump occurring.

[13] This is a technical assumption to guarantee the existence of a stationary firm distribution and equivalent to exogenous death in the Blanchard (1985) and Yaari (1965) model of perpetual youth.

equity which is additionally taxed at the rate η.

Equity holders enjoy limited liability. They can choose to stop paying c in return for surrendering the residual claim to after-tax earnings to the debtholders. They do so strategically by choosing a default strategy that maximizes the value of their claim, trading off tax savings and bankruptcy costs. This behaviour is anticipated by the initial firm owner and taken into account when deciding on the optimal amount of debt to be issued at the firm's inception. As in Gomes and Schmid (2012), firm-level capital structure is static and so to ensure leverage is stationary in the long run, we have exogenous exit as described in Section . When a firm exits, it is replaced by a firm with earnings set to the value X_{0-} with capital structure chosen optimally.

First, we find the prices of levered equity and debt for an exogenously specified coupon rate. Second, we describe an approach to determine the optimal capital structure.

3.3.1 Levered Equity and Optimal Default

We now price the levered equity claim owned by a firm's equityholders. We assume the initial firm owners have decided on capital structure by optimally fixing the coupon level c, so we can focus on pricing the levered equity claim for a fixed coupon rate.

The key decision equityholders make is when to default. We denote the default time by τ_D and equity holders receive the firm's earnings less coupon payments and taxes up until default. Therefore, the time-t price of a firm's levered equity is given by the expected present value of future cashflows less coupon payments up until default, as shown below.

$$S_t = (1-\eta) \sup_{\tau_D \geq t} \tilde{E}_t \left[\int_t^{\tau_D} \frac{\pi_u}{\pi_t} (X_u - c) du \right].$$

The time-t levered equity price will be a function of both current earnings and the current belief that the economy is in a high risk-state, and so $S_t = S(X_t, p_t)$.[14]

If the earnings flow falls temporarily below the coupon rate, c, but the value of levered equity is still positive, equityholders subsidize firm earnings with their own personal wealth so they can still make coupon payments to bondholders and retain ownership of the firm. As soon as the value of levered equity reaches or jumps sharply below zero it will be optimal to declare default, i.e. to give up the claim to the underlying payoff stream and collect a terminal payoff of zero in return.

The optimal stopping problem faced by equityholders leads to the characterization of the default

[14] Proposition B5 shows that $S(p, X)$ satisfies a Hamilton-Jacobi-Bellman Variational Inequality (HJB-VI), allowing us to solve jointly for the the levered equity price and the optimal default boundary. The optimality conditions commonly associated with optimal stopping problems in economics, such as value matching and smooth-pasting (high contact) hold when using the HJB-VI approach, see e.g. Øksendal (2003). Furthermore, Kyprianou and Surya (2007) and Chen and Kou (2009) discuss the validity of using the smooth-pasting principle as an optimality condition when earnings are modeled by various classes of jump processes.

time τ_D as first passage time of the form

$$\tau_D = \inf\left\{t > 0 : X_t \le X_D(p_t)\right\}.$$

The economic impact of learning on the optimal default decision is starkly default from the impact in risk premia. The optimal default threshold rises monotonically as the agent's belief she is in the high-risk state increases (see Figure 8). A lack of noticable curvature implies belief uncertainty does not appreciably impact the optimal default policy in contrast with risk premia.

Default can thus occur in two distinct ways. One, a series of Brownian shocks drags earnings down to touch the default boundary, Two, a downward consumption jump is concurrent with a downward earnings jump while the default boundary jumps upwards as the belief the economy in the high risk-state jumps up. Therefore, learning combined with optimal default amplifies the effects of small downward shifts in consumption on distance-to-default and hence levered equity risk and credit risk. The impact on levered equity is greater than for corporate bonds, and so leverage surges as a series of small jumps in consumption appear.

To understand the conditional levered equity risk premium for a firm, we first consider how a jump in firm-level earnings impacts the levered equity price. We do so by evaluating the expected levered equity return when there is a jump in X without any learning, i.e. $\frac{\tilde{E}_{t-}[S(X_{t-}e^{-Z_{k,t}},p_{t-})|dN_t=1]-S(X_{t-},p_{t-})}{S(X_{t-},p_{t-})} < 0$. We therefore define

$$\overline{J}_X(X,p) = -\frac{\tilde{E}_{t-}[S(Xe^{-Z_{k,t}},p)|dN_t=1] - S(X,p)}{S(X,p)} > 0.$$

Secondly, we consider the impact of learning on the expected levered equity return, but without the impact of the jump in earnings, i.e. $\frac{\tilde{E}_{t-}[S(X_{t-}e^{-Z_{k,t}},p_{t-}[1+J_p(p_{t-})])|dN_t=1]-\tilde{E}_{t-}[S(X_{t-}e^{-Z_{k,t}},p_{t-})|dN_t=1]]}{\tilde{E}_{t-}[S(X_{t-}e^{-Z_{k,t}},p_{t-})|dN_t=1]} < 0$. We therefore define

$$\overline{J}_{pX}(X,p) = -\frac{\tilde{E}_{t-}[S(Xe^{-Z_{k,t}},p[1+J_p(p)])|dN_t=1] - \tilde{E}_{t-}[S(Xe^{-Z_{k,t}},p)|dN_t=1]}{\tilde{E}_{t-}[S(Xe^{-Z_{k,t}},p)|dN_t=1]} > 0$$

Using the above definitions, we can decompose the conditional levered equity risk premium for a firm as shown in the following proposition.

Proposition 2 *The conditional levered equity risk premium for a firm is given by*

$$ERP_t^{lev} = \tilde{E}_{t-}\left[\frac{dR_{X,t}}{dt}\right] - r(p_{t-}) = \Theta_B\sigma_x^{sys}\rho_{cx}\frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}} + \Phi_X(X_{t-},p_{t-}) + \Phi_L(X_{t-},p_{t-}), \quad (14)$$

where the premium for Brownian risks in cashflows is given by $\Theta_B\sigma_x^{sys}\rho_{cx}\frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}} = \gamma\sigma_c\sigma_x^{sys}\rho_{cx}\frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}}$, *the premium for jump-risk in cashflows is given by*

$$\Phi_X(X,p) = (\omega(p)-1)\tilde{\lambda}(p)\overline{J}_X(X,p) > 0$$

and the premium for learning risk is given by

$$\Phi_L(X,p) = (\omega(p)-1)\tilde{\lambda}(p)\overline{J}_{p_X}(X,p)[1-\overline{J}_X(X,p)] > 0.$$

3.3.2 Corporate Bond Prices

We now characterize the price of firm-level corporate debt. The time-t price of perpetual corporate debt issued with coupon rate c is given by

$$D_t = c\tilde{E}_t\left[\int_t^{\tau_D} \frac{\pi_u}{\pi_t} du\right] + \alpha(1-\eta)\tilde{E}_t\left[\frac{\pi_u}{\pi_t} p_X(p_{\tau_D})X_{\tau_D} I_{\{\tau_D \leq \tau_X\}}\right].$$

Debtholders receive a coupon flow c up until endogenous default at the random time τ_D.[15] If endogenous default occurs prior to exogenous exit, then debtholders receive the fraction α of after-tax unlevered firm value. At default, the firm becomes an all equity firm, so firm value is merely the value of unlevered equity, given by $p_X(p_{\tau_D})X_{\tau_D}$. The time-$t$ value of perpetual debt depends on current earnings and current beliefs, i.e. $D_t = D(X_t, p_t)$. The yield on such debt, denoted by $y(X_t, p_t)$ is defined by

$$y(X_t, p_t) = \frac{c}{D(X_t, p_t)}.$$

3.3.3 Optimal Capital Structure

Up to this point, we have discussed how to determine the prices of levered equity, debt and the optimal default boundary for an exogenously specified coupon c. Next we turn to determining the optimal level of debt to be issued at the point of firm inception. We take the perspective of the intial owners of the all equity firm.[16]

Unlike equity owners, initial firms owners care about both $S(X_t, p_t)$ and $D(X_t, p_t)$ when determining their optimal strategy. One can think of them receiving a one-off lump sum payment on the day their firm is founded in return for the debt sold. We additionally assume that the issuance of debt is costly and a fraction of ι deducted from the payment they receive. Thus, the inital firm owners' optimization target becomes $F_{0-} = S_{0-} + (1-\iota)D_{0-}$ and the optimal capital structure problem can be written as

[15] Exogenous exit at the random time τ_X(τ_X is exponentially distributed with parameter λ_k) automatically induces default because earnings must cross the default boundary when earnings jumps to zero.

[16] It is common in literature to tackle the inital firm owners' problem in two distinct steps, see e.g. Leland (1994), Leland and Toft (1996), Goldstein, Ju, and Leland (2001). Once the prices of equity and debt have been found for a given c, one first determines the optimal default response $X_{D,c}$ of equity owners maximizing the value of their claim to firm earnings S_t. Subsequently, one chooses c such that the inital firm owners' wealth is maximized while internalizing the future equity owners' best response Contrary to this, Proposition B5 only takes the coupon level c as given and delivers the optimal default boundary, $X_D(p)$, as part of the solution to the levered equity problem formulated in Section 3.3.1. As a result, the second step is all that remains.

$$(c_{p_{0-},0-}) = \text{argmax}\, F_{0-}\left(c_{p_{0-},0-}, X_{D,c_{p_{0-},0-}}(p_{0-})\right),$$

where the double index $p_{0-,0-}$ underlines the dependence of optimal corporate financial decisions c and $X_D(p_t)$ on the initial state, characterized by an economy wide belief p_{0-} and a firm specific earnings level X_{0-}, in which the firm is launched.[17]

4 Empirics

In this section, we describe data sources and explain how we obtain parameter values for the quantitative evaluation of the model. The continuous-time model is simulated at a daily frequency and time aggregated to a monthly frequency. Firm-level parameters are estimated with the simulated method of moments based on data from 2003 to 2022. Because the model must be solved numerically, estimating all model parameters is computationally infeasible. We, therefore, focus the estimation on those parameters for which the existing literature provides only weak priors – the parameters associated with firms' cash flow risk and bankruptcy costs. The other parameters are based on values in the prior literature.

4.1 Firm Level Data

For the structural estimation, we require data on equity returns, leverage, and CDX prices. Monthly return and leverage data are from CRSP-Compustat. We define the quarterly book value of debt as the sum of short and long-term liabilities (DLCQ plus DLTTQ). Monthly leverage is defined as the most recent quarterly book value of debt divided by the sum of the book value of debt and the market value of equity.

Daily data on credit default swaps (CDS) for the period from September 2003 to June 2022 are obtained from ICE Data Services (formerly known as Credit Market Analysis Ltd. (CMA)). The dataset contains information on pricing (bid and ask quotes) and contract terms of the underlying debt and credit default swaps (e.g., currency, debt seniority, credit event of restructuring, and tenor of the CDS contract).

In this paper, we focus on Markit's North American Investment Grade CDX Index, described in Section 3.1. It is composed of 125 of the most liquid North American firms with investment-grade credit ratings that trade in the CDS market. The index composition is chosen twice annually at

[17] While initial economic conditions have a direct impact on leverage decisions through their impact on e.g. the perceived likelihood of default in the immediate future, the dependence of $X_D(p_t)$ on the coupon level c means they indirectly impact the future equity holders optimal default strategy as well.

times known as roll dates. We match the sample of firms contained in the on-the-run contract with CRSP-Compustat name by name.

For the structural estimation, we target the pooled average and standard deviation of monthly excess returns and leverage for these matched firms. In addition, we target the mean and standard of the 5-year CDX rate as well as the standard deviation of the equal-weighted portfolio return of matched firms.

4.2 Predefined Parameters

Table 3 summarizes predefined parameters. As in Wachter (2013), we assume that the agent has unit elasticity of substitution, which simplifies the discount factor. Wachter (2013) models large infrequent jumps using the distribution of consumption declines found by Barro and Ursua (2008). Since these large infrequent jumps increase the curvature of the stochastic discount factor, she can assume a risk of aversion of 3 for the agent to generate a realistic equity premium. In contrast, we model smaller, more frequent jumps, which generate a realistic distribution of consumption growth, as shown in Section 2.2. To compensate for the smaller curvature in the stochastic discount factor induced by smaller, more frequent jumps, we assume that the representative agent has a risk aversion of 10, as in Bansal and Yaron (2004). We choose the time discount rate β such that the perpetual risk-free yield equals 1%. The correlation between consumption and earning growth is set at 20%, as estimated by Bhamra, Kuehn, and Strebulaev (2010b).

We set the earnings drift μ_x such that the observed net earnings growth rate equals the net consumption growth rate

$$\mu_x = \mu_c - (f_H\lambda_H + f_L\lambda_L)J_c + (f_H\lambda_H + f_L\lambda_L)J_x.$$

In the model, firms issue debt only on the initial date when they are formed. As a result, the importance of leverage vanishes over time as firms outgrow their optimal leverage. To counterbalance these dynamics, firms also face exogenous exit. We set the exogenous exit rate such that the true net earnings growth rate equals zero, which implies that

$$\lambda_x = \mu_c - (f_H\lambda_H + f_L\lambda_L)J_c.$$

As a result, in panel simulations of the model, average leverage is stationary over time.

We set the debt issuance costs at $\iota = 1\%$, based on the empirical evidence in Altinkilic and Hansen (2000) for large firms. Graham (2013) shows in equation (5) that the value of a firm with perpetual debt can be written as

$$F_{\text{with debt}} = F_{\text{no debt}} + \left[1 - \frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_p)}\right] D,$$

where τ_c denotes the corporate tax rate, τ_e the equity payout tax rate, and τ_p the personal tax rate. The term in the square brackets captures the tax advantage of debt. Based on estimates from Kuehn, Schreindorfer, and Schulz (2023), who find that $\tau_c = 0.329$, $\tau_e = 0.112$, and $\tau_p = 0.296$, we set $\eta = 1 - \frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_p)} = 0.154$.

4.3 Estimation

The remaining four model parameters, the amount of idiosyncratic Gaussian risk σ_x^{id}, systematic Gaussian risk σ_x^{sys}, bankruptcy costs $1-\alpha$, and the jump scaling parameter φ, are estimated with the simulated method of moments (SMM) using 7 moments, which are the average of firm-level excess returns, leverage, and 5-year CDX rate, and the standard deviation of firm-level and market excess returns, leverage, and 5-year CDX rate.

Given the predefined parameters summarized in Table 3, and vector $\underline{\theta} = (\sigma_x^{\text{id}}, \sigma_x^{\text{sys}}, \alpha, \varphi)^\top \in \mathbb{R}^4$, we solve the model numerically and simulate panels of firms. The SMM objective function is a weighted metric between seven model moments from simulated panels $\underline{\Psi}^M(\underline{\theta}) \in \mathbb{R}^7$ and the corresponding seven moments from actual data $\underline{\Psi}^D \in \mathbb{R}^7$, defined by the quadratic form $[\underline{\Psi}^D - \underline{\Psi}^M(\underline{\theta})]^\top \mathbf{W} [\underline{\Psi}^D - \underline{\Psi}^M(\underline{\theta})]$, where $\mathbf{W} \in \mathbb{R}^{7\times 7}$ is the seven by seven weighing matrix. Following Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018), we set the diagonal elements of $\mathbf{W}$ to be $(1/\underline{\Psi}^M)^2$ and the off-diagonal elements to zero. Intuitively, with this weighting matrix the SMM estimator minimizes the sum of squared percentage deviations of model moments from the corresponding data moments.

The parameter estimate $\hat{\underline{\theta}}$ is found by searching globally over the parameter space, which we implement via a particle swarm algorithm. Because the numerical model solution is computationally expensive, the estimation requires a high performance computing environment. Computing standard errors for the parameter estimate requires the Jacobian of the moment vector, which we find numerically via a finite difference method.

A noteworthy feature of our 2003 to 2022 sample period is that it contains the Great Recession and the Covid-19 recession. While annual consumption dropped by only 1.6% during the Great Recession, it dropped by 5.1% in 2020. Since this value is small relative to the consumption drop in a typical disaster, it is therefore not appropriate to estimate the model based on the ergodic distribution. Instead, we base the estimation on simulated data that mimics the amount of macroeconomic risk in our sample. Specifically, we simulate 1,000 daily panels of 125 firms over 29 years, where the first

10 years act as burn-in. We then time aggregate daily consumption to annual frequency and ensure that the worst annual consumption drop in each panel does not fall below -5.1%, as in the data. As a result, the (unconditional) consumption growth distribution matches the moments of the post-war sample, as reported in Table 2.

Before explaining the estimation results, we discuss the identification strategy. In Table 4, we report the sensitivity of model-implied moments (in rows) with respect to model parameters (in columns). The sensitivity of moment i with respect to parameter j equals $\frac{\partial \Psi_i^M}{\partial \theta_j} \frac{\theta_j}{\Psi_i^M}$ and is evaluated at the vector of point estimates from Table 5.

Idiosyncratic risk has a large positive impact on the 5-year CDX rate level and dispersion as well as the standard deviation of firm-level returns. The impact of aggregate Brownian risk on the moments is small but it helps to identify the amount of aggregate market risk. Jump risk in earnings are well identified because they impact the level and dispersion of the CDX rate. As jump risk increases, more firms end up in default, leading to a higher cost of credit risk insurance. Bankruptcy costs have a significant negative impact on leverage, as firms optimally delever when they face higher costs of financial distress.

Tables 5 and 6 summarize the estimation. Overall, the model fits the data very well. The model generates an annualized average risk premium of 9.5%, relative to 10.8% in the data. While monthly firm-level returns are very volatile at 9.8% relative to 9.2% in the data, market excess returns are significantly less dispersed at 4.2% relative to 5.1% in the data. The model generates these moments by setting 18.1% idiosyncratic risk and 5.2% aggregate risk.

The model also generates the right amount of leverage of 27.4% relative to 28.9% in the data, and leverage dispersion of 15.3% relative to 15.9% in the data. Given tax shields, the key parameter to identify leverage is the bankruptcy costs. Our estimation implies 34.3% bankruptcy losses. As shown in Chen (2010), time-varying bankruptcy costs alleviate the so-called low leverage puzzle: the typical investment grade firm appears to be under-levered, given the large tax shields and small default probability. With time-varying bankruptcy costs, firms are reluctant to take on leverage not because the deadweight losses of default are high on average, but because the losses are particularly high in those states in which defaults are more likely and losses are more painful. Even though the jump intensity is driven by a Markov switching process in our framework, we cannot tie bankruptcy costs to the Markov state because it is not observable to agents.

Lastly, the model generates a realistic CDX rate for investment grade firms of 72 basis points, relative to 77 basis points in the data. CDX rates are also volatile at 38 basis points relative to 34 basis points in the data. The jump scaling parameter φ is crucial to match credit market moments. Our estimation implies that earnings jumps are 3.3 times higher than consumption jumps. This estimate is in line with data for the Great Depression, where earnings losses was 3.8 times higher than consumption losses.[18]

Statistically, the model is rejected at the 5% level as the p-value of the J-statistics is 2.9%, even though the economic fit is excellent. Interestingly, the point estimate for systematic Brownian shocks is not statistically significant. Our model features two sources of aggregate risk: systematic Brownian shocks, which are correlated with consumption, and Poisson jumps. While earnings losses are larger than consumption losses during disasters, firm-level earnings and consumption jump simultaneously, thereby generating aggregate risk. Empirically, we find that common jump risk is more important for the pricing of credit instruments than common Brownian shocks.

The composition of risk between idiosyncratic and aggregate has a significant impact on credit risk, as shown by Chen, Collin-Dufresne, and Goldstein (2009). Intuitively, if most risks were aggregate, many firms would default at the same time, driving up credit risk. Yet, in the data, the firm-level Sharpe ratio is much smaller than the aggregate Sharpe ratio because individual firms are more volatile than the market. Importantly, our framework can match this fact.

4.4 Term Structure of CDX

In this section, we explore the pricing of the entire term structure of CDX contracts ranging from 1 to 10 years. One can view this exercise as an out-of-sample validation of our model mechanism because we fit the model only to the first and second moments of the 5-year CDX contract, which tends to be the most liquid one.

Figure 10 depicts the term structure of CDX rates in the left panel and the term structure of physical and risk-neutral default probabilities in the right panel. CDX spreads are annual and reported in basis points per unit of notional for contracts with a fixed time to maturity from 1 to 10 years. Model parameters are set to the values reported in Table 1. Empirical averages are computed from daily data on Markits North American Investment Grade CDX Index obtained from ICE Data Services for the period from September 2003 to June 2022. Default probabilities are reported in percent for horizons ranging from 1 to 10 years. Empirical default probabilities are the average cumulative issuer-weighted global default rates reported by Moodys spanning the period from 1920 to 2017 for entities categorized as investment grade (letter rating of Baa3 or better). In Table 7, we report corresponding numerical values.

Even though we only target the first and second moments of the 5-year CDX contract, our model

[18] According to Shiller's stock market data, aggregate real earnings dropped by 62.4% relative to 16.6% for consumption.

does well in matching the entire term structure of CDX rates. In particular, the model generates realistic short-term CDX rates, an average 1-year (3-year) CDX rate of 21 (46) basis points relative to 22 (51) basis points in the data, as well as long-term rates, an average 8-year (10-year) CDX rate of 113 (138) basis points relative to 105 (113) basis points in the data. In contrast, other consumption-based models, such as Bhamra, Kuehn, and Strebulaev (2010b), Chen (2010), and Kuehn, Schreindorfer, and Schulz (2023), can only explain a single point of the term structure.

A resolution to the credit spread puzzle requires a model to match three facts: low leverage, large credit spreads, and low physical default probabilities. Intuitively, market participants are asking for significant compensation for holding credit instruments, although corporate defaults are rare. As shown above, our framework generates realistic leverage and large credit spreads. Figure 10 and Table 7 report the term structure of physical default probabilities, which were not targets of the structural estimation. Overall, our model matches the data very well. At the short end of the term structure, the model implies 1-year (3-year) average physical default probabilities of 0.18% (0.71%) relative to 0.14% (0.72%) in the data, and at the long end 10-year (8-year) 3.71% (2.67%) relative to 3.56% (2.70%) in the data.

Even though physical default probabilities are small, risk-neutral ones are significantly larger, driving up credit spreads. At the short end, risk-neutral default probabilities are 0.31%, which is 1.7 times greater than the corresponding physical one, and at the long end, risk-neutral default probabilities are 17.1%, which is 4.6 times greater than the corresponding physical one. Intuitively, over a 1-year horizon, it is not very likely that the economy enters the high jump risk state, but this risk is rising over longer horizons. As a result, the credit risk compensation is increasing in the term structure.

4.5 Equity Premium

Figure 11 displays the conditional equity risk premium (ERP) of an unlevered firm (left panel) and a levered firm (right panel) for the learning model and full information model, where the agent can observe the state. The dashed red line represents p^*, which is the lower bound for p_t.

For example, for $p_{t-} = 0.36$, the unlevered ERP is about 10%, of which about 4% is due to the learning risk premium, 5.4% is due to the jump risk premium, and the remaining 0.6% is due to Brownian risk. In the case of the unlevered firm, the conditional ERP is only a function of p_{t-}. In contrast to that, time variation in the levered equity premium is caused by both changes in p_{t-} and a firm's earnings levels X_t. In addition, it is also a function of the outstanding coupon chosen at the point of debt issuance. To facilitate a meaningful comparison, we assume that debt was issued when earnings were $X_t = 1$, and the belief about the state was $p_t = p^*$, and we also assume current earnings to be $X_t = 1$. This choice of parameters, while holding the earnings level constant, corresponds to leverage levels ranging from approximately 28% to 35%, depending on the agent's estimate of the current state, thereby keeping it in range of the time series mean reported in Table 6.

In both cases, the ERP is increasing in uncertainty about the state peaking at about $p_{t-} = 0.36$. While the jump risk premium contributes the most at all levels of p_{t-}, the learning risk premium contributes a considerable amount when uncertainty is high, but vanishes as the agent's belief approaches certainty. In absolute terms, the contribution of the Brownian risk premium is constant for the unlevered ERP and varies little in case of the levered ERP. Comparing both panels we can gauge the role of leverage. At an earnings level of 1, the levered ERP is approximately 1.45 times higher independent of the belief held. Moreover, the introduction of leverage does not significantly alter the relative contributions of the different components of the ERP. On the other hand, as leverage approaches zero in the limit as $X_t \to \infty$, the right panel will converge towards the left panel. To complete the comparison, it is instructive to examine the implications for the time series behavior of both ERPs. Consider the sample path of beliefs from Figure 2. Starting from p^*, the belief will jump to about 0.4 after one jump in consumption and earnings is observed, and to higher levels after a sequence of multiple jumps with small interarrival times. This implies that the unlevered ERP will first sharply increase from 4% to 10%, and then start to decrease, reaching a minimum of 5%. In contrast, the direction and magnitude of the change in the levered ERP will crucially depend on the post-jump earnings level, as it is a function of both belief and earnings.

4.6 Time Series Implications

Figure 12 displays the empirical time series of the 5-year maturity spreads for the Markit North American Investment Grade CDX Index and its model-implied equivalent in basis points (top panel), alongside the data on the cross-sectional average CDX leverage in percent (bottom panel), covering the period from September 2003 to June 2022. The average CDX leverage is calculated using available CRSP-Compustat data on book debt and market equity, by referencing the constituent list for each CDX series. To generate the time series for the model-implied CDX, we utilize the leverage time series from the bottom panel and set the unobserved belief to closely align with the empirical time series of CDX spreads. Model parameters are set to the values reported in Table 1.

4.7 Great Depression

The Great Depression is the most severe economic crisis that occurred in the BEA sample of annual consumption data, and it is often used as the exemplar to motivate models with disaster risks. In this section, we explore whether our model can generate dynamics in consumption, equity, and credit prices during simulated Great Depressions, which resemble the data.

To this end, we simulate 10,000 daily consumption paths, time aggregated to annual frequency, where the cumulative drop in annual consumption growth equals approximately 17%, akin to the decline observed during the Great Depression. The simulated data is aligned so that the beginning of a crisis corresponds to time 0, which is the beginning of 1930 in the data. In the following Figures 1, and 13 to 16, we plot the dynamics of model variables averages across 10,000 simulations. It is worth noting that, since we are dating disasters based on annual consumption data, simulations differ in terms of the exact time at which a crisis begins.

In Figure 13, the first panel shows the cross-simulation average of annual cumulative consumption growth from the model and the actual data observed during the Great Depression. Both model and actual data are normalized such that cumulative consumption growth equals one before the first drop in annual consumption occurs.

The second panel displays the average number of jump arrivals in consumption and earnings. The blue bars represent the average number of jumps observed during a particular year, and the red line represents the average cumulative number of jumps, starting at the onset of the observed disaster. Specifically, during the average Great Depression, one would observe 2.7, 2.1, 1.5, and 1.0 jumps during the first to fourth years. Cumulatively, the typical Great Depression features 7.4 jumps.

The third panel displays the average simulated daily time series of beliefs and the average actual state of the Markov chain. Although individual sample paths of both are highly discontinuous, as can be seen in the examples of Figure 2, their cross-sectional averages appear continuous when plotted. For example, during year 0 the Markov chain is in 80% of cases in the high risk state, whereas the belief is only at about 40% as too few jumps have been observed until that point for the belief to have been updated to a higher level.

Figure 14 depicts the average 5-year CDX rate in basis points (first panel), average leverage in percent (second panel) as well as average physical and risk-neutral 5-year default probabilities in percent (third panel). During simulated Great Depressions, credit spreads rise from 135 basis points at the onset of a crisis to 197 basis points one year later, and then peak at 224 basis points in the second year. Similarly, leverage rises from 35% to 44% and 5-year actual default probabilities from

4.5% to 8.5%.

Figure 1 depicts the average loss in the equity market during simulated Great Depressions. The red line represents the daily, cum-dividend, inflation-adjusted cumulative equity return on the CRSP Index observed during the Great Depression. The blue line represents the daily, cum-dividend, cumulative equity return generated by the model, averaged across 125 individual firms and simulations. The shaded areas represent the 50, 80, and 90 percent confidence intervals across simulations.

Figure 16 shows the average dynamics of the conditional equity premium during simulated Great Depressions. The left panel depicts the average daily time series of both the unlevered (red line) and the levered (blue line) conditional equity risk premiums. The right panel illustrates the time series decomposition of the conditional levered equity premium into Brownian, jump, and learning risk premiums, represented by the yellow, purple, and green shadings, respectively.

For example, at the start of year 0 the levered ERP is about 12%, of which about 4.3% is due to the learning risk premium, 6.7% is due to the jump risk premium, and the remaining 1% is due to Brownian risk. The units are in decimals.

5 Conclusion

We have developed a firm-level model of optimal default and capital structure embedded within a consumption-based asset pricing model with an Epstein-Zin representative investor who learns about the stochastic arrival frequency of economic downturns. A sufficiently long sequence of downturns creates a drop in consumption, which can be regarded as a crisis. Consumption dynamics are calibrated to match moments from both an extended sample, including the Great Depression and post-war US data. We perform a structural estimation via the simulated method of moments to estimate parameters relevant to firm-level earnings and corporate financing decisions. Our structural estimation targets the moments of excess levered returns, leverage, and the 5-year CDX rate from 2003-2022. Table 6 shows how successful this targeting is. We also match the joint time series dynamics of leverage and the 5-year CDX rate from 2003-2022, as shown in Figure 12. Figures 14 and 15 show that our model generates realistic dynamics for equity returns, credit risk, and leverage for a Great Depression-like event. Importantly, our model matches the term structure of CDX rates and default rates out-of-sample as shown in Table 7.

Central to our model's empirical prowess are two primary economic underpinnings: firm-specific optimal decisions on default and capital structure, and the Epstein-Zin representative investor's learning about the stochastic arrival frequency of economic downturns. Learning magnifies risk premia and volatilities, but without leverage, risk premia decline as belief uncertainty drops once it becomes clear that the economy is in crisis. Empirically, risk premia rise and remain elevated during crises, as do credit spreads. We show that in contrast to unlevered risk premia, levered risk premia remain elevated despite a drop in belief uncertainty because leverage continues to rise. Leverage rises because levered equity dives in value as the distance-to-default closes. Diminishing earnings play a role in closing the distance-to-default, but so does learning: unlike risk premia, the optimal default boundary does not depend on belief uncertainty – instead, the default boundary increases as it becomes more evident the economy is in crisis.

The successful empirical performance of our model suggests that similar frameworks might improve our understanding of the nexus between asset prices and firm-level real investment, hiring, and risk management decisions.

A Notation

A.1 Consumption, Learning Dynamics, and the Stochastic Discount Factor

$\mathbb{P}$	objective physical probability measure
$E_t[\cdot]$	time-t conditional expectation operator under $\mathbb{P}$
C	stochastic process for aggregate consumption
μ_c	drift of aggregate consumption growth
B_c	standard Brownian motion under $\mathbb{P}$ dB_c is Brownian shock to aggregate consumption growth
σ_c	volatility of aggregate consumption growth driven by Brownian shocks
N	Poisson process under $\mathbb{P}$, dN is Poisson shock to aggregate consumption growth and firm-level earnings growth
λ_t	stochastic intensity of N under $\mathbb{P}$
λ_H	value of the intensity λ in the high risk-state (state H)
λ_L	value of the intensity λ in the low risk-state (state L)
ϕ_{HL}	intensity of transitions from state H to state L under $\mathbb{P}$
ϕ_{LH}	intensity of transitions from state L to state H under $\mathbb{P}$
Z_c	exponentially distributed downward jump in log aggregate consumption
ϵ_c	rate (or inverse scale parameter) for distribution of Z_c under $\mathbb{P}$ $1/\epsilon_c$ is the mean of Z_c under $\mathbb{P}$
$J_c = \frac{1}{1+\epsilon_c}$	mean size of jump in consumption growth under $\mathbb{P}$
$\tilde{\mathbb{P}}$	probability measure representing the representative agent's subjective beliefs
$\widetilde{M}$	exponential martingale under $\mathbb{P}$ used to change measure from $\mathbb{P}$ to $\tilde{\mathbb{P}}$
$\tilde{E}_t[\cdot]$	time-t conditional expectation operator under $\tilde{\mathbb{P}}$
$\mathcal{F}_t$	the σ-algebra representing the representative agent's information set at time-t
$p_t = Pr(\lambda_t = \lambda_H \mid \mathcal{F}_t)$	the representative agent's belief that $\lambda_t = \lambda_H$ conditional on time-t information
$\tilde{\lambda}_t = \tilde{\lambda}(p_t) = p_t\lambda_H + (1-p_t)\lambda_L$	stochastic intensity of N under $\tilde{\mathbb{P}}$
$\kappa = \phi_{LH} + \phi_{HL}$	rate at which λ converges to its long-run mean under $\mathbb{P}$
$f_H = \phi_{LH}/\kappa$	long-run probability under $\mathbb{P}$ that the stochastic intensity of N is in the high risk-state
$f_L = \phi_{HL}/\kappa$	long-run probability under $\mathbb{P}$ that the stochastic intensity of N is in the low risk-state
$\sigma_p(p) = \frac{\lambda_H - \lambda_L}{\tilde{\lambda}(p)} p(1-p)$	representative agent's belief uncertainty
$J_p(p) = \frac{\sigma_p(p)}{p}$	size of jump in the representative agent's beliefs
p^*	attractor of the representative agent's belief in between jumps
J	representative agent's value function
$f(\cdot,\cdot)$	normalized Kreps-Porteus aggregator
β	representative agent's rate of time preference
ψ	representative agent's elasticity of intertemporal consumption
γ	representative agent's relative risk aversion
$V(p_t)$	impact of learning on J measured in units of log consumption
$a(p_{t-}) = V(p_{t-} + \sigma_p(p_{t-})) - V(p_{t-})$	jump in V caused by a jump in p
π	representative agent's equilibrium stochastic discount factor
$r(p)$	equilibrium risk-free rate
$\Theta_B = \gamma\sigma_c$	price of Brownian risk in log aggregate consumption
$\Theta_J(Z_{c,t}) = e^{\gamma Z_{c,t}} - 1$	price of static jump risk in log aggregate consumption
$\Theta_a(p_{t-}, Z_{c,t}) = \Theta_J(Z_{c,t})[e^{-(\gamma-1)a(p_{t-})} - 1]$	price of dynamic learning risk
$\omega(p)$	risk distortion factor
$\lambda^{\mathbb{Q}}(p)$	stochastic intensity of N under $\mathbb{Q}$
M_π	exponential martingale used to change measure from $\tilde{\mathbb{P}}$ to $\mathbb{Q}$

A.2 Firms' Dynamics and CDX Prices

t_{roll}	roll date (date at which the composition of the CDX Index is chosen)
t_{issue}	time at which corporate debt is issued
$\mathcal{N}_F(t_{\text{roll}})$	set of firms in the CDX Index at the roll date t_{roll}
$N_F(t_{\text{roll}})$	number of firms in the CDX Index at the roll date t_{roll}
$D^{\$}_{k,t}$	time-t nominal value of firm k's debt
$\tau_{D,k}$	random default time for firm k's debt
$R^{\$}_{\tau_{D,k}}$	nominal stochastic recovery rate
$n_{t,s}(t_{\text{roll}})$	fraction of firms in the CDX Index defined at the roll date t_{roll}, that have defaulted between the times t and $s > t$
$L^{\$}_{t,s}(t_{\text{roll}})$	sum of the nominal payoffs received by the protection buyer up until time $s > t$
$\text{Prot}^{\$}(T-t)$	nominal present value of the cumulative loss
S_{CDX}	CDX spread
$\text{Prem}(T-t, S_{\text{CDX}})$	nominal present value of cashflows received by the protection seller
X_k	stochastic process for real earnings of firm k
μ_x	drift in real earnings growth rate
σ_x^{id}	idiosyncratic Brownian volatility in real earnings growth
$B_{x,k}$	standard Brownian motion under $\mathbb{P}$ $dB_{x,k}$ is idiosyncratic Brownian shock to earnings growth for firm k
σ_x^{sys}	systematic Brownian volatility in real earnings growth
B_x	standard Brownian motion under $\mathbb{P}$, dB_x is systematic Brownian shock to earnings growth
$\sigma_x = \sqrt{(\sigma_x^{\text{id}})^2 + (\sigma_x^{\text{sys}})^2}$	total Brownian volatility in real earnings growth
Z_k	exponentially distributed jump in log earnings of firm k
ρ_{cx}	correlation between the Brownian shock to consumption growth and systematic Brownian shock to earnings growth
Z_k	exponentially distributed downward jump in log earnings of firm k
ϵ_x	rate (or inverse scale parameter) for distribution of Z_k under $\mathbb{P}$ $1/\epsilon_x$ is the mean of Z_k under $\mathbb{P}$
$\varphi = (1/\epsilon_x)/(1/\epsilon_c)$	scaling parameter for mean log earnings jumps relative to mean log consumption jumps
$J_x = \frac{1}{1+\epsilon_x}$	mean size of jump in earnings growth under $\mathbb{P}$
N_k	Poisson process under $\mathbb{P}$ dN_k is an idiosyncratic Poisson shock which leads to the exit of firm k by driving its earnings to zero
τ_X	random exogenous exit time
η	tax rate
c	coupon rate for debt
p_X	unlevered price-earnings ratio
$J_{p_X}(p)$	size of the jump in the unlevered price-earnings ratio
dR_X^{unlev}	cum-dividend return on unlevered equity
ERP_t^{unlev}	conditional unlevered equity risk premium
$\Pi_X(p)$	unlevered equity risk premium from cashflow jump-risk
$\Pi_L(p)$	unlevered equity risk premium from dynamic learning risk
S	stochastic process for price of levered equity
dR_X	cum-dividend return on levered equity
$\overline{J}_X(X,p)$	size of the expected levered equity return stemming from cashflow jump without learning
$\overline{J}_{p_X}(X,p)$	size of the expected levered equity return stemming from learning without the cashflow jump
$\Phi_X(X,p)$	conditional levered equity risk premium from jump-risk in cashflows
$\Phi_L(X,p)$	conditional levered equity risk premium from dynamic learning risk
$b(p_t)$	time-t price of a perpetual bond which pays one unit of consumption per unit time until a jump in $N_{X,k}$ is realized
D	stochastic process for price of perpetual corporate debt
α	recovery fraction – fraction of after-tax unlevered firm value recovered by debtholders at default
$y(X,p)$	yield on perpetual corporate debt
$X_D(p)$	optimal default boundary as function of p
ι	debt issuance costs
F_{0-}	firm-value at time $0-$ net of debt issuance costs

A.3 Empirics

$F_{\text{with debt}}$	firm value with perpetual debt
$F_{\text{no debt}}$	firm value with no debt
τ_c	corporate tax rate
τ_e	equity payout rate
τ_p	personal rate
$\underline{\theta} = (\sigma_x^{\text{id}}, \sigma_x^{\text{sys}}, \alpha, \varphi)^\top$	vector of predefined parameters
$\hat{\underline{\theta}}$	estimate of $\underline{\theta}$
$\underline{\Psi}^M(\underline{\theta})$	vector of seven model moments
$\underline{\Psi}^D$	vector of seven targeted empirical moments
$\mathbf{W} \in \mathbb{R}^{7\times 7}$	seven by seven weighing matrix used in SMM

A.4 Full Information (No Learning): Value Function, SDF and Equity Returns

$V(\lambda_t)$	impact of changes in the risk-state on J measured in units of log consumption when the risk-state is perfectly observable (no learning)
N_{ij}	Poisson process under $\mathbb{P}$ which jumps up by one when the risk-state changes from from i to $j \neq i$, where $i, j \in \{L, H\}$
$\mathbf{\Phi}$	physical generator matrix for risk-state transitions
$\omega_{ij} - 1$	price of risk for transitions in the risk-state when the current risk-state is i and the risk-state is perfectly observable (no learning)
$\mathbf{\Phi}^{\mathbb{Q}}$	risk-neutral generator matrix for risk-state transitions
k_i	discount rate for unlevered equity in risk-state i when the risk-state is perfectly observable (no learning)
$\mathbf{K} = \text{diag}(k_1, k_2)$	diagonal matrix of discount rates for unlevered equity when the risk-state is perfectly observable (no learning)
$r_{i,x}$	discount rate for perpetual risk-free debt in risk-state i when the risk-state is perfectly observable (no learning)
$\mathbf{R}_x = \text{diag}(r_{i,x}, r_{i,x})$	diagonal matrix of discount rates for perpetual risk-free debt when the risk-state is perfectly observable (no learning)
$\sigma_{R,i,t-}$	conditional volatility of levered equity returns in risk-state i when the risk-state is perfectly observable (no learning)

B Proofs

Proof. Equation (2) is the counterpart of the Wonham filter[19] for the case where updating is based on observing a jump process instead of a continuous-path process – the filter can be obtained as a special case of Theorem 19.6, page 332 of Liptser and Shiryaev (2013), and is used in Benzoni, Collin-Dufresne, and Goldstein (2011) and Wachter and Zhu (2019). Between jump times (2) reduces to a Ricatti ordinary differential equation given by

$$\frac{dp_t}{dt} = (\lambda_H - \lambda_L)\left[\overline{\kappa}(f_H - p_t) - p_t(1-p_t)\right],$$

which can be rewritten as

$$\frac{dp_t}{dt} = (\lambda_H - \lambda_L)(p_t - \overline{p})(p_t - p^*), \tag{B1}$$

where

$$\overline{p} = \frac{1+\overline{\kappa}}{2}\left[1 + \sqrt{1 - \frac{4f_H\overline{\kappa}}{(1+\overline{\kappa})^2}}\right], \tag{B2}$$

$$p^* = \frac{1+\overline{\kappa}}{2}\left[1 - \sqrt{1 - \frac{4f_H\overline{\kappa}}{(1+\overline{\kappa})^2}}\right], \tag{B3}$$

$$\overline{\kappa} = \frac{\kappa}{\lambda_H - \lambda_L}.$$

We now show that $p^\star$ is the unique steady state solution of (B1). We do so by showing that $\overline{p} > 1$ and $p^\star \in (0,1)$. From (B2) and the fact that $f_H \in (0,1)$, we see that

$$\begin{aligned}\overline{p} &> \frac{1+\overline{\kappa}}{2}\left[1 + \sqrt{1 - \frac{4\overline{\kappa}}{(1+\overline{\kappa})^2}}\right]\\ &= \frac{1}{2}\left[(1+\overline{\kappa}) + \sqrt{(1-\overline{\kappa})^2}\right].\end{aligned}$$

We have either $\overline{\kappa} > 1$, $\overline{\kappa} = 1$, or $\overline{\kappa} < 1$. If $\overline{\kappa} > 1$, then $\sqrt{(1-\overline{\kappa})^2} = \overline{\kappa} - 1$, and so $\overline{p} > \overline{\kappa} > 1$. If $\overline{\kappa} = 1$, then $\sqrt{(1-\overline{\kappa})^2} = 0$, and so $\overline{p} > (1+\overline{\kappa})/2 = 1$. If $\overline{\kappa} < 1$, then $\sqrt{(1-\overline{\kappa})^2} = 1 - \overline{\kappa}$, and so $\overline{p} > 1$. Therefore $\overline{p} > 1$.

Similarly, via (B3) and and the fact that $f_H \in (0,1)$, we see that

$$p^* < \frac{1+\overline{\kappa}}{2}\left[1 - \sqrt{1 - \frac{4\overline{\kappa}}{(1+\overline{\kappa})^2}}\right],$$

from which it follows that $p^* < 1$. It follows immediately from (B3) and $f_H \in (0,1)$ that $p^* > 0$. Therefore, $p^\star \in (0,1)$.

If the last jump time is denoted by τ, then we can solve the Ricatti differential equation (B1) to obtain the following solution for beliefs between jump times:

$$p_t = p^* + \frac{1}{(p_\tau - p^*)^{-1}e^{(\lambda_H-\lambda_L)\Delta(t-\tau)} + (\lambda_H - \lambda_L)^{-1}[1 - e^{(\lambda_H-\lambda_L)\Delta(t-\tau)}]},$$

where τ is the most recent jump time, $t > \tau$, and

$$\Delta = (1+\overline{\kappa})\sqrt{1 - \frac{4f_H\overline{\kappa}}{(1+\overline{\kappa})^2}}.$$

■

Proposition B1 *The representative agent's stochastic discount factor is given by*

$$\pi(C_t, p_t) = \beta e^{-\beta\int_0^t[1-(\gamma-1)V(p_s)]ds} C_t^{-\gamma} e^{-(\gamma-1)V(p_t)},$$

where for $p \in [0,1]$, $V(p)$ satisfies the functional differential equation

$$0 = \mu_p(p)V'(p) + \tilde{\lambda}(p)\frac{\frac{1-J_c}{1-\gamma J_c}e^{(1-\gamma)\left[V\left(\left(\frac{\lambda_H}{\lambda(p)}\right)p\right) - V(p)\right]} - 1}{1-\gamma} - \beta V(p) + \mu_c - \frac{1}{2}\gamma\sigma_c^2, \tag{B4}$$

where

$$\mu_p(p) = \kappa(f_H - p) - (\lambda_H - \lambda_L)p(1-p), \tag{B5}$$

where at the internal point $p^* \in (0,1)$, given by (B3), we have

$$0 = \tilde{\lambda}(p^*)\frac{\frac{1-J_c}{1-\gamma J_c}e^{(1-\gamma)\left[V\left(\left(\frac{\lambda_H}{\lambda(p^*)}\right)p^*\right) - V(p^*)\right]} - 1}{1-\gamma} - \beta V(p^*) + \mu_c - \frac{1}{2}\gamma\sigma_c^2. \tag{B6}$$

[19] See Wonham (1964) and applications in David (1997b), Veronesi (1999), and Veronesi (2000).

Proof of Proposition B1. We derive the functional differential equation for $V(p)$, given in (B4). Let J_t be the value function for exogenous aggregate consumption as defined in (4), the Feynman-Kac theorem implies

$$f(C_{t-},J_{t-})dt+\tilde{E}_{t-}\left[dJ(C_t,p_t)\right]=0.$$

Using Ito's Lemma we rewrite the above equation as

$$0=f\left(C_{t-},J_{t-}\right)+\mu_cC_{t-}J_{t-,C}+\frac{1}{2}\sigma_c^2C_{t-}^2J_{t-,CC}+\mu_p(p_{t-})J_{t-,p}+\tilde{\lambda}(p_{t-})\tilde{E}_{t-}\left[J(C_t,p_t)-J(C_{t-},p_{t-})|dN_t=1\right], \quad \text{(B7)}$$

where $\mu_p(\cdot)$ is defined in (B5). Note that when $dN_t=1$ both arguments change in $J(C,p)$. We guess and verify that the representative agent's value function is given by (6). From (1) we know that if a jump occurs at t, then

$$C_t=C_{t-}(1+(e^{-Z_{c,t}}-1))=C_{t-}e^{-Z_{c,t}},$$

while from (2) we can see that

$$p_t=p_{t-}\left(1+\frac{\lambda_H-\tilde{\lambda}(p_{t-})}{\tilde{\lambda}(p_{t-})}\right)=\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}p_{t-}.$$

Together with (6), we can now see that

$$\begin{aligned}J(C_t,p_t)-J(C_{t-},p_{t-})&=\frac{(C_{t-}e^{-Z_{c,t}})^{1-\gamma}}{1-\gamma}e^{(1-\gamma)V\left(\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}p_{t-}\right)}-J(C_{t-},p_{t-})\\&=J(C_{t-},p_{t-})\left[e^{(1-\gamma)\left[-Z_{c,t}+V\left(\left(\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)p_{t-}\right)-V(p_{t-})\right]}-1\right].\end{aligned}$$

Substituting (6) and (B8) into (B7) yields

$$0=\mu_c-\frac{1}{2}\gamma\sigma_c^2-\beta V(p_{t-})+\mu_p(p_{t-})V'(p_{t-})+\tilde{\lambda}(p_{t-})\frac{E_{t-}[e^{-(1-\gamma)Z_{c,t}}|dN_t=1]e^{(1-\gamma)\left[V\left(\left(\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)p_{t-}\right)-V(p_{t-})\right]}-1}{1-\gamma}, \quad \text{(B9)}$$

where $\mu_p(p)$ is defined in (B5). We know that $Z_{c,t}$ is negative exponentially distributed with mean $1/\epsilon_c>0$ under $\mathbb{P}$ (and hence $\tilde{\mathbb{P}}$). The probability density function for $Z_{c,t}$ is given by $p(x)=\epsilon_ce^{-\epsilon_cx}$, $x\geq 0$, and so

$$E_{t-}\left[e^{-\delta Z_{c,t}}|dN_t=1\right]=\tilde{E}_{t-}\left[e^{-\delta Z_{c,t}}|dN_t=1\right]=\int_0^\infty\epsilon_ce^{-\epsilon_cx-\delta x}dx=\int_0^\infty\epsilon_ce^{-(\epsilon_c+\delta)x}dx=\frac{\epsilon_c}{\epsilon_c+\delta}=\frac{1-J_c}{1-(1-\delta)J_c}, \quad \text{(B10)}$$

which implies

$$E_{t-}\left[e^{-(1-\gamma)Z_{c,t}}|dN_t=1\right]=\tilde{E}_{t-}\left[e^{-(1-\gamma)Z_{c,t}}|dN_t=1\right]=\frac{1-J_c}{1-\gamma J_c}, \quad \text{(B11)}$$

where J_c is the average drop in consumption due to a jump, i.e. $\epsilon_c=\frac{1}{J_c}-1$. Therefore, (B9) can be written as

$$0=\mu_c-\frac{1}{2}\gamma\sigma_c^2-\beta V(p_{t-})+\mu_p(p_{t-})V'(p_{t-})+\tilde{\lambda}(p_{t-})\frac{\frac{1-J_c}{1-\gamma J_c}e^{(1-\gamma)\left[V\left(\left(\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)p_{t-}\right)-V(p_{t-})\right]}-1}{1-\gamma},$$

and so, setting $p_{t-}=p$ gives the functional differential equation (B4). We set $p=p^*$ in (B4) to obtain the condition (B6).
■

Proof of Proposition 1.

We now derive the dynamics of the SDF. Duffie and Skiadas (1994) show that the SDF for a general normalized aggregator f is given by

$$\pi_t=e^{\int_0^t f_v(C_s,J_s)ds}f_c\left(C_t,J_t\right),$$

where $f_c(\cdot,\cdot)$ and $f_v(\cdot,\cdot)$ are the partial derivatives of f with respect to its first and second arguments, respectively, and J is the value function given in (4). Thus, taking the derivatives of (5) and substituting (6) we obtain

$$\pi_t=\beta e^{-\beta\int_0^t[1+(1-\gamma)V(p_s)]ds}C_t^{-\gamma}e^{(1-\gamma)V(p_t)}. \quad \text{(B12)}$$

Applying Ito's Lemma we obtain

$$\begin{aligned}d\pi_t&=\frac{\partial\pi_t}{\partial t}dt+C_{t-}\frac{\partial\pi_t}{\partial C_t}(\mu_cdt+\sigma_cdB_{c,t})+\frac{\partial\pi_t}{\partial p_t}\mu_p(p_{t-})dt+\frac{1}{2}C_{t-}^2\frac{\partial^2\pi_t}{\partial C_t^2}\sigma_c^2dt+\left(e^{\gamma Z_{c,t}+(1-\gamma)a(p_{t-})}-1\right)dN_t\\\frac{d\pi_t}{\pi_{t-}}&=-\beta[1+(1-\gamma)V(p_{t-})]-\gamma(\mu_cdt+\sigma_cdB_{c,t})+(1-\gamma)V'(p_{t-})\mu_p(p_{t-})dt+\frac{1}{2}\gamma(1+\gamma)\sigma_c^2dt+\left(e^{\gamma Z_{c,t}+(1-\gamma)a(p_{t-})}-1\right)dN_t\end{aligned}$$

and so

$$\frac{d\pi_t}{\pi_{t-}} = -\kappa(p_{t-})dt - \gamma\sigma_c dB_{c,t} + \left(e^{\gamma Z_{c,t}+(1-\gamma)a\left(p_{t-}\right)} - 1\right)dN_t,$$

where $a\left(p_{t-}\right)$ is defined in (7) and $\kappa(p_{t-})$ is given by the following function

$$\kappa(p) = \beta + \gamma\mu_c - \frac{1}{2}\gamma(1+\gamma)\sigma_c^2 + (\gamma-1)\left(\mu_p(p)V'(p) - \beta V(p)\right).$$

We now use the functional differential equation (B4) to eliminate the dependence of $\kappa(p)$ on $V'(p)$, i.e.

$$\begin{aligned}
\kappa(p) &= \beta + \gamma\mu_c - \frac{1}{2}\gamma(1+\gamma)\sigma_c^2 - (\gamma-1)\left(\mu_c - \frac{1}{2}\gamma\sigma_c^2 + \tilde{\lambda}(p_t)\frac{\frac{1-J_c}{1-\gamma J_c}e^{(1-\gamma)\left[V\left(\left(\frac{\lambda_H}{\tilde{\lambda}(p)}\right)p\right)-V(p)\right]} - 1}{1-\gamma}\right), \\
&= \beta + \gamma\mu_c - \frac{1}{2}\gamma(1+\gamma)\sigma_c^2 - (\gamma-1)\left(\mu_c - \frac{1}{2}\gamma\sigma_c^2 + \tilde{\lambda}(p_t)\frac{\frac{1-J_c}{1-\gamma J_c}e^{(1-\gamma)a(p)} - 1}{1-\gamma}\right), \\
&= \beta + \mu_c - \gamma\sigma_c^2 + \tilde{\lambda}(p_t)\left(\frac{1-J_c}{1-\gamma J_c}e^{(1-\gamma)a(p)} - 1\right).
\end{aligned}$$

Now, in the absence of arbitrage, we have

$$\tilde{E}_{t-}[d\pi_t] = -r_{t-}\pi_{t-}dt,$$

where r_{t-} is the time-$t-$ risk-free rate. Therefore,

$$\begin{aligned}
\frac{d\pi_t}{\pi_{t-}} &= -r_{t-}dt - \gamma\sigma_c dB_{c,t} + \left(e^{\gamma Z_{c,t}+(1-\gamma)a\left(p_{t-}\right)} - 1\right)dN_t - \tilde{\lambda}(p_{t-})\left(E_{t-}[e^{\gamma Z_{c,t}}|dN_t = 1]e^{(1-\gamma)a\left(p_{t-}\right)} - 1\right)dt \\
&= -r_{t-}dt - \gamma\sigma_c dB_{c,t} + \left(e^{\gamma Z_{c,t}+(1-\gamma)a\left(p_{t-}\right)} - 1\right)dN_t - \tilde{\lambda}(p_{t-})\left(\frac{1-J_c}{1-J_c(1+\gamma)}e^{(1-\gamma)a\left(p_{t-}\right)} - 1\right)dt,
\end{aligned}$$

and $r_{t-} = r(p_{t-})$, where

$$r(p) = \kappa(p) - \left(\frac{1-J_c}{1-J_c(1+\gamma)}e^{(1-\gamma)a(p)} - 1\right)\tilde{\lambda}(p),$$

and we have used the fact that $\tilde{E}_{t-}[e^{\gamma Z_{c,t}}|dN_t = 1] = \frac{1-J_c}{1-J_c(1+\gamma)}$. Using the definition of the risk distortion factor in (12), we see that

$$\frac{d\pi_t}{\pi_{t-}} = -r_{t-}dt - \gamma\sigma_c dB_{c,t} + \left(e^{\gamma Z_{c,t}+(1-\gamma)a\left(p_{t-}\right)} - 1\right)dN_t - \tilde{\lambda}(p_{t-})\left(\omega(p_{t-}) - 1\right)dt. \tag{B13}$$

In order to distinguish the impact of jumps in beliefs from jumps in consumption, we rewrite (B13) as

$$\frac{d\pi_t}{\pi_{t-}} = -r_{t-}dt - \gamma\sigma_c dB_{c,t} + \left[e^{\gamma Z_{c,t}} - 1 + e^{\gamma Z_{c,t}}\left(e^{(1-\gamma)a\left(p_{t-}\right)} - 1\right)\right]dN_t - \tilde{\lambda}(p_{t-})\left(\omega(p_{t-}) - 1\right)dt. \tag{B14}$$

We now define the stochastic process M_π as the solution to the following stochastic differential equation under $\tilde{\mathbb{P}}$

$$\frac{dM_{\pi,t}}{M_{\pi,t-}} = \left[e^{\gamma Z_{c,t}}e^{(1-\gamma)a\left(p_{t-}\right)} - 1\right]dN_t - \tilde{\lambda}(p_{t-})\left(\omega(p_{t-}) - 1\right)dt, M_{\pi,0} = 1.$$

We see that M_π is an exponential martingale under $\tilde{\mathbb{P}}$ and defines the change of measure from $\tilde{\mathbb{P}}$ to $\mathbb{Q}$, and so the risk-neutral intensity of N, denoted by $\lambda^{\mathbb{Q}}(p_{t-})$, is given by

$$\lambda^{\mathbb{Q}}(p_{t-}) = E_{t-}^{\mathbb{Q}}\left[\frac{dN_t}{dt}\right] = \tilde{E}_{t-}\left[\frac{M_{\pi,t}}{M_{\pi,t-}}\frac{dN_t}{dt}\right],$$

where, of course, $\tilde{E}_{t-}[M_{\pi,t}] = M_{\pi,t-}$. We now evaluate $\tilde{E}_{t-}\left[\frac{M_{\pi,t}}{M_{\pi,t-}}dN_t\right]$.

$$\begin{aligned}
\tilde{E}_{t-}\left[\frac{M_{\pi,t}}{M_{\pi,t-}}dN_t\right] &= \tilde{E}_{t-}\left[\left\{1 + \left[e^{\gamma Z_{c,t}}e^{(1-\gamma)a\left(p_{t-}\right)} - 1\right]dN_t - \tilde{\lambda}(p_{t-})\left(\omega(p_{t-}) - 1\right)dt\right\}dN_t\right] \\
&= \tilde{E}_{t-}\left[\left\{1 + \left[e^{\gamma Z_{c,t}}e^{(1-\gamma)a\left(p_{t-}\right)} - 1\right] - \tilde{\lambda}(p_{t-})\left(\omega(p_{t-}) - 1\right)dt\right\}|dN_t = 1\right]\tilde{\lambda}(p_{t-})dt \\
&= \tilde{E}_{t-}\left[e^{\gamma Z_{c,t}}|dN_t = 1\right]e^{(1-\gamma)a\left(p_{t-}\right)}\tilde{\lambda}(p_{t-})dt + o(dt).
\end{aligned}$$

Hence,

$$\lambda^{\mathbb{Q}}(p_{t-}) = \tilde{E}_{t-}\left[e^{\gamma Z_{c,t}}|dN_t = 1\right] e^{(1-\gamma)a(p_{t-})}\tilde{\lambda}(p_{t-})$$
$$= \frac{1-J_c}{1-(1+\gamma)J_c} e^{(1-\gamma)a(p_{t-})}\tilde{\lambda}(p_{t-}).$$

To distinguish between the impact of static jump risk versus dynamic learning risk, we rewrite the above expression as

$$\lambda^{\mathbb{Q}}(p_{t-}) = \tilde{\lambda}(p_{t-})\left[\frac{1-J_c}{1-(1+\gamma)J_c} + \frac{1-J_c}{1-(1+\gamma)J_c}\left(e^{(1-\gamma)a(p_{t-})} - 1\right)\right].$$

We hence obtain (11). We can now rewrite (B14) as (8).
■

The unlevered price-earnings ratio of a firm is given by

$$p_{X,t} = \tilde{E}_t\left[\int_t^\infty \frac{\pi_s}{\pi_t}\frac{X_s}{X_t}ds\right],$$

where we omit the subscript k for ease of notation.

The unlevered price-earnings ratio will depend on the representative agent's belief that the economy is in the high-risk state, i.e. $p_{X,t} = p_X(p_t)$. The risk premium on a firm's unlevered price is its unlevered equity risk premium.

Proposition B2 *The conditional unlevered equity risk premium for a firm is given by*

$$ERP_t^{unlev} = \tilde{E}_{t-}\left[\frac{dR_{X,t}^{unlev}}{dt} - r(p_{t-})\right] = \Theta_B\sigma_x^{sys}\rho_{cx} + \Pi_X(p_{t-}) + \Pi_L(p_{t-}), \tag{B15}$$

where the premium for Brownian risks in cashflows is given by $\Theta_B\sigma_x^{sys}\rho_{cx} = \gamma\sigma_c\sigma_x^{sys}\rho_{cx}$*, the premium for jump-risk in cashflows is given by*

$$\Pi_X(p) = (\omega(p) - 1)\tilde{\lambda}(p)J_x > 0$$

and the premium for learning risk is given by

$$\Pi_L(p) = (\omega(p) - 1)\tilde{\lambda}(p)(1 - J_x)J_{p_X}(p) > 0,$$

where

$$J_{p_X}(p) = -\left(\frac{p_X\left(p[1 + J_p(p)]\right)}{p_X(p)} - 1\right) > 0.$$

Proposition B3 *The price-earnings ratio* $p_X(p)$ *solves the functional differential equation*

$$0 = \mu(p)p_X'(p) - (r(p) + \lambda_x + \gamma\sigma_c\sigma_x^{sys}\rho_{cx} - \mu_x)\, p_X(p) + \omega(p)\tilde{\lambda}(p)\left[p_X\left(p[1 + J_p(p)]\right)(1 + J_x) - p_X(p)\right] + 1, \tag{B16}$$

with condition

$$0 = -\left(r(p^*) + \lambda_x + \gamma\sigma_c\sigma_x^{sys}\rho_{cx} - \mu_x\right) p_X(p^*) + \omega(p^*)\tilde{\lambda}(p^*)\left[p_X\left(p^*[1 + J_p(p^*)]\right)(1 + J_x) - p_X(p^*)\right] + 1 \tag{B17}$$

Proof of Propositions B2 and B3.

The cum-dividend return on a firm's unlevered equity is denoted by $dR_{X,t}^{\text{unlev}}$, where, using Ito's Lemma, we obtain

$$\begin{aligned}
dR_{X,t}^{\text{unlev}} &= \frac{p_X'(p_t)}{p_X(p_{t-})}\mu_p(p_{t-})dt + \mu_x dt + \sigma_x^{\text{id}}dB_{x,k,t} + \sigma_x^{\text{sys}}dB_{x,t} + \frac{1}{p_X(p_{t-})}dt \\
&+ \frac{\left[p_X\left(p_{t-}\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)X_{t-}e^{-Z_{k,t}} - p_X(p_{t-})X_{t-}\right]dN_t - p_X(X_{t-})X_{t-}dN_{k,t}}{p_X(X_{t-})X_{t-}} \\
&= \frac{p_X'(p_t)}{p_X(p_{t-})}\mu_p(p_{t-})dt + \mu_x dt + \sigma_x^{\text{id}}dB_{x,k,t} + \sigma_x^{\text{sys}}dB_{x,t} + \frac{1}{p_X(p_{t-})}dt + \left[\frac{p_X\left(p_{t-}\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)e^{-Z_{k,t}}}{p_X(p_{t-})} - 1\right]dN_t - dN_{k,t} \\
&= \left(\mu_x + \frac{p_X'(p_{t-})}{p_X(p_{t-})}\mu(p_{t-}) + \frac{1}{p_X(p_{t-})}\right)dt + \sigma_x^{\text{id}}dB_{x,k,t} + \sigma_x^{\text{sys}}dB_{x,t} - dN_{k,t} + \left[\frac{p_X\left(p_{t-}\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)e^{-Z_{k,t}}}{p_X(p_{t-})} - 1\right]dN_t.
\end{aligned}$$

Therefore, the conditional expected cum-dividend return is given by

$$\tilde{E}_{t-}[dR^{\text{unlev}}_{X,t}] = \left(\mu_x + \frac{p'_X(p_t)}{p_X(p_{t-})}\mu(p_{t-}) + \frac{1}{p_X(p_{t-})}\right)dt - \lambda_x dt + \left[\frac{p_X\left(p_{t-}\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)}{p_X(p_{t-})}\tilde{E}_{t-}\left[e^{-Z_{k,t}}\right] - 1\right]\tilde{\lambda}(p_{t-})dt$$

$$= \left(\mu_x + \frac{p'_X(p_{t-})}{p_X(p_{t-})}\mu(p_{t-}) + \frac{1}{p_X(p_{t-})}\right)dt - \lambda_x dt + \left[\frac{p_X\left(p_{t-}\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)}{p_X(p_{t-})}\frac{\epsilon_x}{1+\epsilon_x} - 1\right]\tilde{\lambda}(p_{t-})dt,$$

and

$$-\tilde{E}_{t-}\left[dR^{\text{unlev}}_{X,t}\frac{d\pi_t}{\pi_{t-}}\right] = \gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx}dt - \left[\frac{p_X\left(p_{t-}\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)}{p_X(p_{t-})}\frac{\epsilon_x}{1+\epsilon_x} - 1\right](\omega(p_{t-})-1)\tilde{\lambda}(p_{t-})dt$$

Thus, from the basic asset pricing equation

$$\tilde{E}_{t-}\left[dR^{\text{unlev}}_{X,t} - r(p_{t-})dt\right] = -\tilde{E}_{t-}\left[dR^{\text{unlev}}_{X,t}\frac{d\pi_t}{\pi_{t-}}\right],$$

we see that the conditional unlevered equity risk premium for a firm is

$$\tilde{E}_{t-}\left[\frac{dR^{\text{unlev}}_{X,t}}{dt} - r(p_{t-})\right] = \gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx} - (\omega(p_{t-})-1)\tilde{\lambda}(p_{t-})\left[\frac{p_X\left(p_{t-}\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)}{p_X(p_{t-})}\frac{\epsilon_x}{1+\epsilon_x} - 1\right],$$

which we can rewrite as (B15). We also obtain the following functional differential equation

$$0 = \mu_x + \frac{p'_X(p)}{p_X(p)}\mu(p) + \frac{1}{p_X(p)} - \lambda_x + \left[\frac{p_X\left(p\frac{\lambda_H}{\tilde{\lambda}(p)}\right)}{p_X(p)}\frac{\epsilon_x}{1+\epsilon_x} - 1\right]\tilde{\lambda}(p) - r(p) = \gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx}$$

$$- (\omega(p)-1)\tilde{\lambda}(p)\left[\frac{p_X\left(p\frac{\lambda_H}{\tilde{\lambda}(p)}\right)}{p_X(p)}\frac{\epsilon_x}{1+\epsilon_x} - 1\right],$$

and so

$$0 = \mu(p)\frac{p'_X(p)}{p_X(p)} + \frac{1}{p_X(p)} - (r(p) + \lambda_x + \gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx} - \mu_x) + \omega(p)\tilde{\lambda}(p)\left[\frac{p_X\left(p\frac{\lambda_H}{\tilde{\lambda}(p)}\right)}{p_X(p)}\frac{\epsilon_x}{1+\epsilon_x} - 1\right],$$

which we can rewrite as

$$0 = \mu(p)p'_X(p) - (r(p) + \lambda_x + \gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx} - \mu_x)\,p_X(p) + \omega(p)\tilde{\lambda}(p)\left[p_X\left(p\frac{\lambda_H}{\tilde{\lambda}(p)}\right)\frac{\epsilon_x}{1+\epsilon_x} - p_X(p)\right] + 1,$$

from which we obtain (B16). Setting $p = p^*$, we obtain the following condition

$$0 = -(r(p^*) + \lambda_x + \gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx} - \mu_x)\,p_X(p^*) + \omega(p^*)\tilde{\lambda}(p^*)\left[p_X\left(p^*\frac{\lambda_H}{\tilde{\lambda}(p^*)}\right)\frac{\epsilon_x}{1+\epsilon_x} - p_X(p^*)\right] + 1,$$

which gives (B17). ■

We define

$$\frac{dc_{k,t}}{c_{k,t}} = -dN_{X,k,t},$$

meaning that c stays fixed until a jump in $N_{X,k}$ is realized. We now consider a perpetual bond which pays one unit of consumption per unit time until a jump in $N_{X,k}$ is realized. The time-t price of such a bond is denoted by $b(p_t)$, where

$$b(p_t) = \tilde{E}_t\left[\int_t^\infty \frac{\pi_s}{\pi_t}\frac{c_s}{c}\,ds\right],$$

and we omit the k subscript for ease of notation.

Proposition B4 *The time-t price of a perpetual bond which pays one unit of consumption per unit time until a jump in $N_{X,k}$ is realized is denoted by $b(p_{t-})$, where $b(p)$ satisfies the following functional ordinary differential equation*

$$0 = \mu(p)b'(p) - (r(p) + \lambda_x)\, b(p) + \omega(p)\tilde{\lambda}(p)\left[b\left(p[1 + J_p(p)]\right)(1 + J_x) - b(p)\right] + 1, \tag{B19}$$

with condition

$$0 = -\left(r(p^*) + \lambda_x\right) b(p^*) + \omega(p^*)\tilde{\lambda}(p^*)\left[b\left(p^*[1 + J_p(p^*)]\right)(1 + J_x) - b(p^*)\right] + 1. \tag{B20}$$

Proof of Proposition B4. The instantaneous return on the bond, including the coupon flow, is given by

$$\frac{db(p_{t-}) + dt}{b(p_{t-})} = \left(\frac{b'(p_{t-})}{b(p_{t-})}\mu(p_{t-}) + \frac{1}{b(p_{t-})}\right)dt - dN_{k,t} + \left[\frac{b\left(p_{t-}\frac{\lambda_H}{\tilde{\lambda}(p_{t-})}\right)e^{-Z_{k,t}}}{b(p_{t-})} - 1\right]dN_t.$$

From the principle of no arbitrage, we obtain

$$\tilde{E}_t\left[\frac{db(p_{t-}) + dt}{b(p_{t-})} - r(p_{t-})dt\right] = -\tilde{E}_{t-}\left[\frac{d\pi_t}{\pi_{t-}}\frac{db(p_{t-})}{b(p_{t-})}\right]$$

Therefore, we obtain

$$0 = \mu(p)b'(p) - (r(p) + \lambda_x)\, b(p) + \omega(p)\tilde{\lambda}(p)\left[b\left(p\frac{\lambda_H}{\tilde{\lambda}(p)}\right)\frac{\epsilon_x}{1 + \epsilon_x} - b(p)\right] + 1,$$

from which we obtain (B19). Setting $p = p^*$, we obtain the following condition

$$0 = \omega(p^*)\tilde{\lambda}(p^*)\left[b\left(p^*\frac{\lambda_H}{\tilde{\lambda}(p^*)}\right)\frac{\epsilon_x}{1 + \epsilon_x} - b(p^*)\right] - (r(p^*) + \lambda_x)\, b(p^*) + 1,$$

which gives (B20). We solve for $b(p)$ via Chebyshev collocation. ■

Proposition B5 *The price of levered equity $S(X_t, p_t)$ satisfies the following Hamilton-Jacobi-Bellman Variational Inequality (HJB-VI):*

$$\begin{aligned} min\Big\{&(\lambda_x + r(p))S(X,p) - (1-\eta)(X - c) - (\mu_x - \gamma\sigma_c\sigma_x^{sys}\rho_{cx})XS_X(X,p) - \mu(p)S_p(X,p) \\ &-\frac{1}{2}X^2\sigma_x^2 S_{XX}(X,p) - \tilde{E}_{t-}[S(Xe^{-Z_{k,t}}, p(1 + J(p))) - S(X,p)|dN_t = 1]\tilde{\lambda}(p)\omega(p), S(X,p)\Big\} = 0, \end{aligned}$$

with boundary conditions

$$\lim_{X\to 0} S(X,p) = 0$$

and

$$\lim_{X\to\infty} S(X,p) = (1-\eta)\left(p_X(p)X - b(p)c\right).$$

Proof of Proposition B5. The cum-dividend return on a firm's levered equity is given by $dR_t = \frac{dS_t}{S_{t-}} + \frac{(1-\eta)(X_t - c)}{S_{t-}}dt$, and so using Ito's Lemma, we obtain

$$\begin{aligned} dR_t &= \frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}}\left(\mu_x dt + \sigma_x^{\mathrm{id}}dB_{x,k,t} + \sigma_x^{\mathrm{sys}}dB_{x,t}\right) + \frac{1}{S_{t-}}\frac{\partial S_{t-}}{\partial p_t}\mu_p(p_{t-})dt + \frac{(1-\eta)(X_{t-} - c)}{S_{t-}}dt \\ &+ \left[S(X_{t-}e^{-Z_{k,t}}, p_{t-}[1 + J_p(p_{t-})]) - S(X_{t-}, p_{t-})\right]dN_t + \left[0 - S(X_{t-}, p_{t-})\right]dN_{k,t} + \frac{1}{2}\frac{X_{t-}^2}{S_{t-}}\frac{\partial^2 S_{t-}}{\partial X_{t-}^2}\sigma_x^2 dt \\ &= \left(\frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}}\mu_x + \frac{1}{S_{t-}}\frac{\partial S_{t-}}{\partial p_t}\mu_p(p_{t-}) + \frac{1}{2}\frac{X_{t-}^2}{S_{t-}}\frac{\partial^2 S_{t-}}{\partial X_{t-}^2}\sigma_x^2 + \frac{(1-\eta)(X_{t-} - c)}{S_{t-}}\right)dt + \frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}}\left(\sigma_x^{\mathrm{id}}dB_{x,k,t} + \sigma_x^{\mathrm{sys}}dB_{x,t}\right) \\ &+ \left[\frac{S(X_{t-}e^{-Z_{k,t}}, p_{t-}[1 + J_p(p_{t-})])}{S(X_{t-}, p_{t-})} - 1\right]dN_t - dN_{k,t} \end{aligned}$$

where

$$\sigma_x^2 = \left(\sigma_x^{\mathrm{id}}\right)^2 + \left(\sigma_x^{\mathrm{sys}}\right)^2.$$

Therefore, the conditional expected cum-dividend levered equity return is given by

$$\begin{aligned}\tilde{E}_{t-}[dR_t] &= \left(\frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}}\mu_x + \frac{1}{S_{t-}}\frac{\partial S_{t-}}{\partial p_t}\mu_p(p_{t-}) + \frac{1}{2}\frac{X_{t-}^2}{S_{t-}}\frac{\partial^2 S_{t-}}{\partial X_{t-}^2}\sigma_x^2 + \frac{(1-\eta)(X_{t-}-c)}{S_{t-}}\right)dt + \\ &+ \left\{\frac{\tilde{E}_{t-}[S(X_{t-}e^{-Z_{k,t}}, p_{t-}[1+J_p(p_{t-})])|dN_t = 1]}{S(X_{t-}, p_{t-})} - 1\right\}\tilde{\lambda}(p_{t-})dt - \lambda_x dt\end{aligned}$$

and

$$-\tilde{E}_{t-}\left[dR_t\frac{d\pi_t}{\pi_{t-}}\right] = \gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx}\frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}}dt - \left\{\frac{\tilde{E}_{t-}[S(X_{t-}e^{-Z_{k,t}}, p_{t-}[1+J_p(p_{t-})])|dN_t = 1]}{S(X_{t-}, p_{t-})} - 1\right\}\tilde{\lambda}(p_{t-})(\omega(p_{t-})-1)dt$$

Thus, from the basic asset pricing equation

$$\tilde{E}_{t-}\left[dR_t - r(p_{t-})dt\right] = -\tilde{E}_{t-}\left[dR_t\frac{d\pi_t}{\pi_{t-}}\right],$$

we see that the conditional levered equity risk premium for a firm is

$$\tilde{E}_{t-}\left[\frac{dR_t}{dt}\right] - r(p_t) = \gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx}\frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}} - \left\{\frac{\tilde{E}_{t-}[S(X_{t-}e^{-Z_{k,t}}, p_{t-}[1+J_p(p_{t-})])|dN_t = 1]}{S(X_{t-}, p_{t-})} - 1\right\}\tilde{\lambda}(p_{t-})(\omega(p_{t-})-1),$$

which we can rewrite as (14)

We also obtain the following functional partial differential equation

$$\begin{aligned}&\frac{X_{t-}}{S_{t-}}\frac{\partial S_{t-}}{\partial X_{t-}}(\mu_x - \gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx}) + \frac{1}{S_{t-}}\frac{\partial S_{t-}}{\partial p_t}\mu_p(p_{t-}) + \frac{1}{2}\frac{X_{t-}^2}{S_{t-}}\frac{\partial^2 S_{t-}}{\partial X_{t-}^2}\sigma_x^2 + \frac{(1-\eta)(X_{t-}-c)}{S_{t-}} - (r(p_{t-}) + \lambda_x) \\ &= -\left\{\frac{\tilde{E}_{t-}[S(X_{t-}e^{-Z_{k,t}}, p_{t-}[1+J_p(p_{t-})])|dN_t = 1]}{S(X_{t-}, p_{t-})} - 1\right\}\tilde{\lambda}(p_{t-})\omega(p_{t-}),\end{aligned}$$

which is valid when the firm is not in default, i.e. $X_{t-} > X_D(p_{t-})$. When $X_{t-} \leq X_D(p_{t-})$, we have $S(X_{t-}, p_{t-}) = 0$. Hence, we obtain (B21). ■

C Full Information (No Learning): Value Function, SDF and Equity Returns

Proposition C6 *When the value of the transition intensity λ_t is always known, then the representative agent's value function is given by*

$$J(C_t, \lambda_t) = h(e^{V(\lambda_t)}C_t), \tag{C1}$$

where

$$V(\lambda_L) = \frac{\mu_c - \frac{1}{2}\gamma\sigma_c^2 - \lambda_L \frac{J_c}{1-\gamma J_c} + \phi_{LH}\frac{\omega_\lambda - 1}{1-\gamma}}{\beta}, \tag{C2}$$

$$V(\lambda_H) = \frac{\mu_c - \frac{1}{2}\gamma\sigma_c^2 - \lambda_H \frac{J_c}{1-\gamma J_c} + \phi_{HL}\frac{\omega_\lambda^{-1} - 1}{1-\gamma}}{\beta}, \tag{C3}$$

and $\omega_\lambda > 0$ is given by the nonlinear algebraic equation (C8), which has a unique solution if and only if $\beta^2 > 4\phi_{LH}\phi_{HL}$.

Proof of Proposition C6. The representative agent's value function is of the form (C1), where the function $V(\lambda_t)$ captures how the (physical) intensity of jumps in consumption impacts the agent's utility. For ease of notation, we shall write

$$J(C_t, \lambda_i) = J_i(C_t) = h(e^{V_i}C_t),\ i \in \{L, H\}$$

where

$$V_i = V(\lambda_i),\ i \in \{L, H\}.$$

We now derive a system of nonlinear algebraic equations for V_L and V_H. With exogenous aggregate consumption the Feynman-Kac theorem implies

$$f(C_{t-}, J_i)dt + E_{t-}\left[dJ_i(C_{t-})|\lambda_{t-} = \lambda_i\right] = 0.$$

Using Ito's Lemma we rewrite the above equation as

$$0 = f\left(C_{t-}, J_i(C_{t-})\right) + \mu_c C_{t-}J_i(C_{t-}) + \frac{1}{2}\sigma_c^2 C_{t-}^2 J_{i,CC}(C_{t-}) + \lambda_i\left(E_{t-}\left[J_i(C_t)|dN_t = 1\right] - J_i(C_{t-})\right) + \phi_{ij}\left[J_j(C_{t-}) - J_i(C_{t-})\right],\ j \neq i \tag{C4}$$

We guess and verify that the representative agent's value function is given by (C1). From (1) we know that if a jump occurs at t, then

$$C_t = C_{t-}(1 + (e^{-Z_{c,t}} - 1)) = C_{t-}e^{-Z_{c,t}},$$

Together with (C1), we can now see that

$$\begin{aligned} E_{t-}\left[J_i(C_t)|dN_t = 1\right] - J_i(C_{t-})) &= J_i(C_{t-})\left(E_{t-}\left[\frac{J_i(C_t)}{J_i(C_{t-})}|dN_t = 1\right] - 1\right) \\ &= J_i(C_{t-})\left(E_{t-}[e^{-(1-\gamma)Z_c}|dN_t = 1] - 1\right). \end{aligned}$$

Using (B11), we hence obtain

$$E_{t-}\left[J_i(C_t)|dN_t = 1\right] - J_i(C_{t-}) = J_i(C_{t-})\left[\frac{1 - J_c}{1 - \gamma J_c} - 1\right] = J_i(C_{t-})(\gamma - 1)\frac{J_c}{1 - \gamma J_c}. \tag{C6}$$

Substituting (C1) and (C6) into (C4) yields the following system of nonlinear algebraic equations for V_L and V_H:

$$0 = \mu_c - \frac{1}{2}\gamma\sigma_c^2 - \beta V_i - \lambda_i\frac{J_c}{1 - \gamma J_c} + \phi_{ij}\frac{e^{(1-\gamma)(V_j - V_i)} - 1}{1 - \gamma},\ i, j \in \{L, H\},\ j \neq i. \tag{C7}$$

Therefore, we obtain the following nonlinear algebraic equation for $V_L - V_H$

$$0 = -\beta(V_L - V_H) - (\lambda_L - \lambda_H)\frac{J_c}{1 - \gamma J_c} + \phi_{LH}\frac{e^{(1-\gamma)(V_H - V_L)} - 1}{1 - \gamma} - \phi_{HL}\frac{e^{(1-\gamma)(V_L - V_H)} - 1}{1 - \gamma}.$$

Defining

$$\omega_\lambda = e^{(\gamma - 1)(V_L - V_H)},$$

we obtain the following nonlinear algebraic equation for ω_λ

$$0 = \beta\ln\omega_\lambda + (\gamma - 1)(\lambda_H - \lambda_L)\frac{J_c}{1 - \gamma J_c} + \phi_{LH} - \phi_{HL} - \phi_{LH}\omega_\lambda + \phi_{HL}\omega_\lambda^{-1}. \tag{C8}$$

By definition $\omega_\lambda > 0$. We now prove that for $\omega_\lambda > 0$ the above equation has a unique real solution if and only if $\beta^2 < 4\phi_{LH}\phi_{HL}$. We define

$$h(x) = \beta \ln x + (\gamma - 1)(\lambda_H - \lambda_L)\frac{J_c}{1 - \gamma J_c} + \phi_{LH} - \phi_{HL} - \phi_{LH}x + \phi_{HL}x^{-1},\; x > 0,$$

which is a continuous function. Therefore, $h(x)$ is monotonically decreasing if and only if $h'(x) < 0$ for $x > 0$. Now

$$h'(x) = \frac{\beta}{x} - \phi_{LH} - \frac{\phi_{HL}}{x^2}, x > 0.$$

and so $h'(x) < 0$ for $x > 0$ if and only if

$$\phi_{LH}x^2 - \beta x + \phi_{HL} > 0 \,\text{for}\, x > 0.$$

We know that $\phi_{LH} > 0$ and $\beta > 0$, and so the condition will be satisfied if and only if the roots of the quadratic $\phi_{LH}x^2 - \beta x + \phi_{HL}$ are complex. By computing the discriminant of the quadratic, we can see that the roots are complex if and only if $\beta^2 < 4\phi_{LH}\phi_{HL}$, yielding the required result.

From (C7) we can now see that

$$V_L = \frac{\mu_c - \frac{1}{2}\gamma\sigma_c^2 - \lambda_L\frac{J_c}{1-\gamma J_c} + \phi_{LH}\frac{\omega_\lambda - 1}{1-\gamma}}{\beta},$$
$$V_H = \frac{\mu_c - \frac{1}{2}\gamma\sigma_c^2 - \lambda_H\frac{J_c}{1-\gamma J_c} + \phi_{HL}\frac{\omega_\lambda^{-1} - 1}{1-\gamma}}{\beta}.$$

■

Proposition C7 *When the value of the transition intensity λ_t is always known, the dynamics of the equilibrium SDF are given by*

$$\frac{d\pi_t}{\pi_{t-}}|_{\lambda_{t-}=\lambda_i} = -r(\lambda_i)dt - \Theta_B dB_{c,t} + \left(e^{\gamma Z_{c,t}} - 1\right)dN_t + (\omega_{ij} - 1)dN_{ij,t},\; j \neq i,\; i,j \in \{L, H\}, \tag{C9}$$

where the risk-free rate $r(\lambda_i) = r_i$ is given by

$$r_i = \beta + \mu_c - \gamma\sigma_c^2 - \lambda_i\frac{1 - J_c}{1 - (\gamma+1)J_c}\frac{J_c}{1 - \gamma J_c}, \tag{C10}$$
$$= \beta + \mu_c - \gamma\sigma_c^2 - \lambda_i^{\mathbb{Q}}\frac{J_c}{1 - \gamma J_c}, \tag{C11}$$

and the risk-neutral jump intensity is given by

$$\lambda_t^{\mathbb{Q}} = \lambda_t\frac{1 - J_c}{1 - (\gamma+1)J_c}. \tag{C12}$$

The price of jump-risk is given by $e^{\gamma Z_c} - 1$ and $\omega_{ij} - 1$ is the price of risk for transitions in the risk-state when the current risk-state is i, where

$$\omega_{LH} = \omega_\lambda,\; \omega_{HL} = \omega_\lambda^{-1}.$$

The risk-neutral transition intensities for risk-states are given by $\phi_{ij}^{\mathbb{Q}}$, $j \neq i$, $i,j \in \{L, H\}$, where

$$\phi_{ij}^{\mathbb{Q}} = \omega_{ij}\phi_{ij}.$$

N_{ij} is a Poisson process under $\mathbb{P}$ which jumps up by one when the risk-state changes from from i to $j \neq i$, where $i,j \in \{L, H\}$.

Proof of Proposition C7. We now derive the dynamics of the SDF. By adapting (B12), we obtain

$$\pi_t = \beta e^{-\beta\int_0^t[1+(1-\gamma)V(\lambda_s)]ds}C_t^{-\gamma}e^{(1-\gamma)V(\lambda_t)}.$$

Applying Ito's Lemma we obtain

$$\begin{aligned} d\pi_t &= \frac{\partial \pi_t}{\partial t}dt + C_{t-}\frac{\partial \pi_t}{\partial C_t}(\mu_c dt + \sigma_c dB_{c,t}) + \frac{1}{2}C_{t-}^2\frac{\partial^2 \pi_t}{\partial C_t^2}\sigma_c^2 dt + \left(e^{\gamma Z_{c,t}} - 1\right)dN_t \\ &\quad + \left[\beta e^{-\beta\int_0^t[1+(1-\gamma)V(\lambda_s)]ds}C_{t-}^{-\gamma}e^{(1-\gamma)V_j} - \beta e^{-\beta\int_0^t[1+(1-\gamma)V(\lambda_s)]ds}C_{t-}^{-\gamma}e^{(1-\gamma)V_i}\right]dN_{ij,t} \\ \frac{d\pi_t}{\pi_{t-}} &= -\beta[1 + (1-\gamma)V(\lambda_{t-})]dt - \gamma(\mu_c dt + \sigma_c dB_{c,t}) + \frac{1}{2}\gamma(1+\gamma)\sigma_c^2 dt + \left(e^{\gamma Z_{c,t}} - 1\right)dN_t \\ &\quad + \left[e^{(1-\gamma)(V_j - V_i)} - 1\right]dN_{ij,t}, \end{aligned} \tag{C13}$$

where N_{ij} is a Poisson process under $\mathbb{P}$ which jumps up by one when the risk-state changes from from i to $j \neq i$, where $i, j \in \{L, H\}$. We substitute the expressions from (C2) and (C3) into (C13) to obtain

$$\frac{d\pi_t}{\pi_{t-}} = -r(\lambda_{t-})dt - \gamma\sigma_c dB_{c,t} + \left(e^{\gamma Z_{c,t}} - 1\right) dN_t + (\omega_{ij} - 1)dN_{ij,t},\ j \neq i,\ i, j \in \{L, H\},$$

where

$$r(\lambda_t) = \beta + \mu_c - \gamma\sigma_c^2 - \lambda_t \frac{(1 - J_c)J_c}{(1 - \gamma J_c)(1 - (\gamma + 1)J_c)},$$

giving (C10). The risk-neutral jump intensity is given by

$$\begin{aligned}\lambda_{t-}^{\mathbb{Q}} &= \lambda_{t-} E_{t-}\left[e^{\gamma Z_{c,t}}\right] \\ &= \lambda_{t-} \frac{1 - J_c}{1 - (1 + \gamma)J_c},\end{aligned}$$

and so we obtain (C11) and (C12). With Θ_B defined as in (9), we obtain (C9) from (C13). ∎

Proposition C8 *When the jump transition intensity, λ_t, is always known the time-t unlevered price-dividend ratio is given by*

$$p_{X,t} = p_X(\lambda_t),$$

where $p_X(\lambda_L) = p_{X,L}$ and a $p_X(\lambda_H) = p_{X,H}$ are the elements of the vector $\underline{p}_X = (p_{X,L}, p_{X,H})^\top$, given by

$$\underline{p}_X = (\mathbf{K} - \mathbf{\Phi}^{\mathbb{Q}})^{-1}\underline{1}. \tag{C14}$$

The 2 by 2 matrix $\mathbf{K}$ is the diagonal matrix of discount rates given by

$$\mathbf{K} = diag(k_1, k_2), \tag{C15}$$

where

$$k_i = r_i + \lambda_x + \gamma\sigma_c\sigma_x^{sys}\rho_{cx} + \lambda_i^{\mathbb{Q}} J_x + \phi_{ij}^{\mathbb{Q}} - \mu_x,\ i, j \in \{L, H\},\ j \neq i. \tag{C16}$$

The 2 by 2 matrix $\mathbf{\Phi}^{\mathbb{Q}}$ is the risk-neutral generator matrix for risk-state transitions given by

$$[\mathbf{\Phi}^{\mathbb{Q}}]_{ij} = \begin{cases} -\sum_{k\in\{L,H\}-\{i\}} \phi_{ik}^{\mathbb{Q}} & , i = j \\ \phi_{ij}^{\mathbb{Q}} & , i \neq j \end{cases}. \tag{C17}$$

$\underline{1} = (1, 1)^\top$ is the 2 by 1 vector of ones. The unlevered risk premium is given by

$$E_{t-}\left[\frac{dR_{X,i,t}^{unlev}}{dt} - r(\lambda_{t-})|\lambda_{t-} = \lambda_i\right] = \gamma\sigma_c\sigma_x^{sys}\rho_{cx} + (\lambda_i^{\mathbb{Q}} - \lambda_i)J_x - (\phi_{ij}^{\mathbb{Q}} - \phi_{ij})\frac{p_{X,j} - p_{X,i}}{p_{X,i}}\ i, j \in \{L, H\},\ j \neq i. \tag{C18}$$

Proof of Proposition C8. The time-t unlevered price-dividend ratio depends on λ_t, which is known, and so

$$p_{X,t} = p_X(\lambda_t).$$

For ease of notation, we define

$$p_{X,L} = p_X(\lambda_L),\ p_{X,H} = p_X(\lambda_H).$$

Conditional on the current risk-state being $i \in \{L, H\}$, the cum-dividend return on a firm's unlevered equity is denoted by $dR_{X,i,t}^{\text{unlev}}$, where, using Ito's Lemma, we obtain

$$dR_{X,i,t}^{\text{unlev}} = \mu_x dt + \sigma_x^{\text{id}} dB_{x,k,t} + \sigma_x^{\text{sys}} dB_{x,t} + \frac{1}{p_{X,i}}dt + (e^{-Z_{k,t}} - 1)dN_t - dN_{k,t} + \frac{p_{X,j} - p_{X,i}}{p_{X,i}}dN_{ij,t}$$

Therefore, the conditional expected cum-dividend return is given by

$$\begin{aligned}E_{t-}[dR_{X,i,t}^{\text{unlev}}] &= \mu_x dt + + \frac{1}{p_{X,i}}dt + \lambda_i(E_t[e^{-Z_{k,t}}|dN_t = 1] - 1) - \lambda_x dt + \phi_{ij}\frac{p_{X,j} - p_{X,i}}{p_{X,i}}dt \\ &= \mu_x dt + + \frac{1}{p_{X,i}}dt - \lambda_i \frac{1}{1 + \epsilon_x} - \lambda_x dt + \phi_{ij}\frac{p_{X,j} - p_{X,i}}{p_{X,i}}dt,\end{aligned}$$

and

$$-E_{t-}\left[dR^{\text{unlev}}_{X,i,t}\frac{d\pi_t}{\pi_{t-}}|\lambda_{t-}=\lambda_i\right]=\gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx}dt-\lambda_i E_{t-}[e^{\gamma Z_{c,t}}-1|dN_t=1]E_{t-}[e^{-Z_{k,t}}-1|dN_t=1]dt-\phi_{ij}(\omega_{ij}-1)\frac{p_{X,j}-p_{X,i}}{p_{X,i}}dt,$$

where we have exploited the independence of $Z_{c,t}$ and $Z_{k,t}$. We now use the same calculation as in (B10) to see that

$$E_{t-}[e^{-Z_{k,t}}|dN_t=1]=\frac{\epsilon_x}{\epsilon_x+1}.$$

Using the above result and (B11), we see that

$$-E_{t-}\left[dR^{\text{unlev}}_{X,i,t}\frac{d\pi_t}{\pi_{t-}}|\lambda_{t-}=\lambda_i\right]=\gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx}dt+\lambda_i\frac{J_c\gamma}{1-J_c(1+\gamma)}J_x dt-\phi_{ij}(\omega_{ij}-1)\frac{p_{X,j}-p_{X,i}}{p_{X,i}}dt.$$

Hence, we obtain

$$E_{t-}\left[dR^{\text{unlev}}_{X,i,t}\frac{d\pi_t}{\pi_{t-}}|\lambda_{t-}-\lambda_i\right]=\gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx}dt+(\lambda_i^{\mathbb{Q}}-\lambda_i)J_x dt-(\phi_{ij}^{\mathbb{Q}}-\phi_{ij})\frac{p_{X,j}-p_{X,i}}{p_{X,i}}dt.$$

Thus, from the basic asset pricing equation

$$E_{t-}\left[dR^{\text{unlev}}_{X,i,t}-r(\lambda_{t-})dt|\lambda_{t-}=\lambda_i\right]=-E_{t-}\left[dR_{X,i,t}\frac{d\pi_t}{\pi_{t-}}|\lambda_{t-}=i\right],$$

we see that the conditional unlevered equity risk premium for a firm is

$$E_{t-}\left[\frac{dR^{\text{unlev}}_{X,i,t}}{dt}-r(\lambda_{t-})|\lambda_{t-}=\lambda_i\right]=\gamma\sigma_c\sigma_x^{\text{sys}}\rho_{cx}+(\lambda_i^{\mathbb{Q}}-\lambda_i)J_x-(\phi_{ij}^{\mathbb{Q}}-\phi_{ij})\frac{p_{X,j}-p_{X,i}}{p_{X,i}},$$

which we can rewrite as (C18). We also obtain the following linear equation system

$$\phi_{ij}^{\mathbb{Q}}p_{X,j}-k_i p_{X,i}+1=0,\ j\neq i,\ i,j\in\{L,H\},$$

where k_i is defined in (C16). We now write the above system in vector-matrix form as follows

$$(\mathbf{K}-\mathbf{\Phi}^{\mathbb{Q}})\underline{p}_X=\underline{1},$$

where $\mathbf{K}$ is defined in (C15), $\mathbf{\Phi}^{\mathbb{Q}}$ is defined in (C19), $\underline{p}_X=(p_{X,L},p_{X,H})^\top$, and $\underline{1}=(1,1)^\top$. It follows that $\underline{p}_X$ is given by (C14). [20] ■

Proposition C9 *When the jump transition intensity, λ_t, is always known the time-t perpetual bond price is given by*

$$b_t=b(\lambda_t),$$

where $b(\lambda_L)=b_L$ and a $b(\lambda_H)=b_H$ are the elements of the vector $\underline{b}=(b_L,b_H)^\top$, given by

$$\underline{b}=c(\mathbf{R}_x-\mathbf{\Phi}^{\mathbb{Q}})^{-1}\underline{1}.$$

The 2 by 2 matrix $\mathbf{R}_x$ is the diagonal matrix of discount rates given by

$$\mathbf{R}_x=diag(r_{1,x},r_{2,x}),$$

where

$$r_{i,x}=r_i+\lambda_x,\ i,j\in\{L,H\},\ j\neq i.$$

The 2 by 2 matrix $\mathbf{\Phi}^{\mathbb{Q}}$ is the risk-neutral generator matrix for risk-state transitions given by

$$[\mathbf{\Phi}^{\mathbb{Q}}]_{ij}=\begin{cases}-\sum_{k\in\{L,H\}-\{i\}}\phi_{ik}^{\mathbb{Q}} & ,\ i=j\\ \phi_{ij}^{\mathbb{Q}} & ,\ i\neq j\end{cases}. \tag{C19}$$

$\underline{1}=(1,1)^\top$ is the 2 by 1 vector of ones.

[20] The 2 by 2 matrix $\mathbf{\Phi}$ is the physical generator matrix for risk-state transitions given by

$$[\mathbf{\Phi}]_{ij}=\begin{cases}-\sum_{k\in\{L,H\}-\{i\}}\phi_{ik} & ,\ i=j\\ \phi_{ij} & ,\ i\neq j\end{cases}.$$

Proof of Proposition C9. Proposition C9 is a special case of Proposition C8 obtained by setting cashflow risk to zero. ■

Proposition C10 *The conditional levered equity risk premium for a firm is given by*

$$E_{t-}\left[\frac{dR_{X,i,t}}{dt}-r(\lambda_{t-})|\lambda_{t-}=\lambda_i\right]=\gamma\sigma_c\sigma_x^{sys}\rho_{cx}\frac{X_{t-}S_X(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}-\frac{E_{t-}\left[S(X_{t-}e^{-Z_{k,t}},\lambda_i)\right]-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}\left(\lambda_i^{\mathbb{Q}}-\lambda_i\right)$$
$$-\frac{S(X_{t-},\lambda_j)-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}(\phi_{ij}^{\mathbb{Q}}-\phi_{ij}),\ i,j\in\{L,H\},\ j\neq i.$$

Proof of Proposition C10. The time-t levered stock price is denoted by $S(X_t,\lambda_t)$, where λ_t is known and can take two values: λ_L and λ_H. The levered return is denoted by $dR_{X,t}$, where

$$dR_{X,i,t}=dR_{X,t}|_{\lambda_{t-}=\lambda_i}=\frac{X_{t-}S_X(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}\left[\mu_x dt+\sigma_x^{\mathrm{id}}dB_{x,k,t}+\sigma_x^{\mathrm{sys}}dB_{x,t}\right]+\frac{(1-\eta)(X_t-c)}{S(X_{t-},\lambda_i)}dt+\frac{S(X_{t-}e^{-Z_{k,t}},\lambda_i)-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}dN_t$$
$$-dN_{k,t}+\frac{S(X_{t-},\lambda_j)-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}dN_{ij,t},\ i,j\in\{L,H\},\ j\neq i.$$

The conditional volatility of levered equity returns is given by

$$\sigma_{R,i,t-}=\sqrt{\left(\frac{S_X(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}\right)^2\sigma_x^2+\lambda_k+E_{t-}\left[\left(\frac{S(X_{t-}e^{-Z_{k,t}},\lambda_i)-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}\right)^2\right]\lambda_i+\left(\frac{S(X_{t-},\lambda_j)-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}\right)^2\phi_{ij}}.$$

The conditional expected levered cum-dividend return is given by

$$E_{t-}[dR_{X,i,t}]=\mu_x\frac{S_X(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}dt+\frac{(1-\eta)(X_t-c)}{S(X_{t-},\lambda_i)}dt+\frac{E_{t-}\left[S(X_{t-}e^{-Z_{k,t}},\lambda_i)\right]-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}\lambda_i dt-\lambda_x dt$$
$$+\phi_{ij}\frac{S(X_{t-},\lambda_j)-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}dt$$

and

$$-E_{t-}\left[dR_{X,i,t}\frac{d\pi_t}{\pi_{t-}}|\lambda_{t-}=\lambda_i\right]=\gamma\sigma_c\sigma_x^{\mathrm{sys}}\rho_{cx}\frac{S_X(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}dt-\frac{E_{t-}\left[S(X_{t-}e^{-Z_{k,t}},\lambda_i)\right]-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}\left(E_{t-}[e^{\gamma Z_{c,t}}]-1\right)\lambda_i dt$$
$$-\phi_{ij}(\omega_{ij}-1)\frac{S(X_{t-},\lambda_j)-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}dt$$

Thus, from the basic asset pricing equation

$$E_{t-}\left[dR_{X,t}-r(\lambda_{t-})dt|\lambda_{t-}=\lambda_i\right]=-E_{t-}\left[dR_{X,i,t}\frac{d\pi_t}{\pi_{t-}}|\lambda_{t-}=i\right],$$

we see that the conditional levered equity risk premium for a firm is

$$E_{t-}\left[\frac{dR_{X,i,t}}{dt}-r(\lambda_{t-})|\lambda_{t-}=\lambda_i\right]=\gamma\sigma_c\sigma_x^{\mathrm{sys}}\rho_{cx}\frac{X_{t-}S_X(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}-\frac{E_{t-}\left[S(X_{t-}e^{-Z_{k,t}},\lambda_i)\right]-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}\left(\lambda_i^{\mathbb{Q}}-\lambda_i\right)$$
$$-\frac{S(X_{t-},\lambda_j)-S(X_{t-},\lambda_i)}{S(X_{t-},\lambda_i)}(\phi_{ij}^{\mathbb{Q}}-\phi_{ij}),$$

and so for $X\geq X_{D,i}$

$$0=(\mu_x-\gamma\sigma_c\sigma_x^{\mathrm{sys}})S_X(X,\lambda_i)+\left[E_{t-}[S(Xe^{-Z_{k,t}},\lambda_i)]-S(X,\lambda_i)\right]\lambda_i^{\mathbb{Q}}+[S(X,\lambda_j)-S(X,\lambda_i)]\phi_{ij}^{\mathbb{Q}}+(1-\eta)(X-c)-(r(\lambda_i)+\lambda_x)S(X,\lambda_i).$$

We thus obtain the following HJB-VI:

$$\min\Big\{(r(\lambda_i)+\lambda_x)S(X,\lambda_i)-(1-\eta)(X-c)-(\mu_x-\gamma\sigma_c\sigma_x^{\mathrm{sys}}\rho_{cx})XS_X(X,\lambda_i)-[S(X,\lambda_j)-S(X,\lambda_i)]\phi_{ij}^{\mathbb{Q}}$$
$$-[S(X,\lambda_j)-S(X,\lambda_i)]\phi_{ij}^{\mathbb{Q}}-\left[E_{t-}[S(Xe^{-Z_{k,t}},\lambda_i)]-S(X,\lambda_i)\right]\lambda_i^{\mathbb{Q}}),S(X,\lambda)\Big\}=0,$$

with boundary conditions

$$\lim_{X\to 0}S(X,\lambda)=0$$

and

$$\lim_{X\to\infty}S(X,\lambda)=(1-\eta)\left(p_X(\lambda)X-b(\lambda)c\right).$$

■

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Figure 2: Learning Dynamics

This figure illustrates simulated sample paths for the unobserved Markov chain, the belief p_t that the Markov chain is currently in a high-risk state, the Markov modulated Poisson process N_t, and its time-varying intensity λ_t. Model parameters are set to the values reported in Table 1. Across all panels, the hidden Markov chain transitions into the high jump risk state in year 10 and exits this state in year 14, as indicated by the grey background shading. The top right panel depicts the sample path of N_t. As the Markov chain switches regimes, the frequency of jumps in N_t changes notably. The top left panel illustrates the evolution of the belief over time. The belief is updated in accordance with the observation of jumps (or the lack thereof) in N_t according to (2). Between jump arrivals, the belief continuously decays towards the lower bound p^*, represented by the dashed line. On the other hand, with each arrival in N_t, the belief increases sharply. The bottom panel compares the true value of the λ_t with its estimated value $\tilde{\lambda}(p_t) = \lambda_L\ (1 - p_t) + \lambda_H\ p_t$. As the Markov chain enters the high-risk state, the true intensity increases nearly tenfold, leading to a notable increase in the frequency of observed jump arrivals in N_t and, consequently, a rapid upward adjustment of the belief. Eventually, the estimated $\tilde{\lambda}(p_t)$ approaches the true λ_t , albeit with a delay. Conversely, after the switch in year 14, the arrival frequency drastically declines, causing the belief to decay steadily and the estimate of λ_t to decrease.

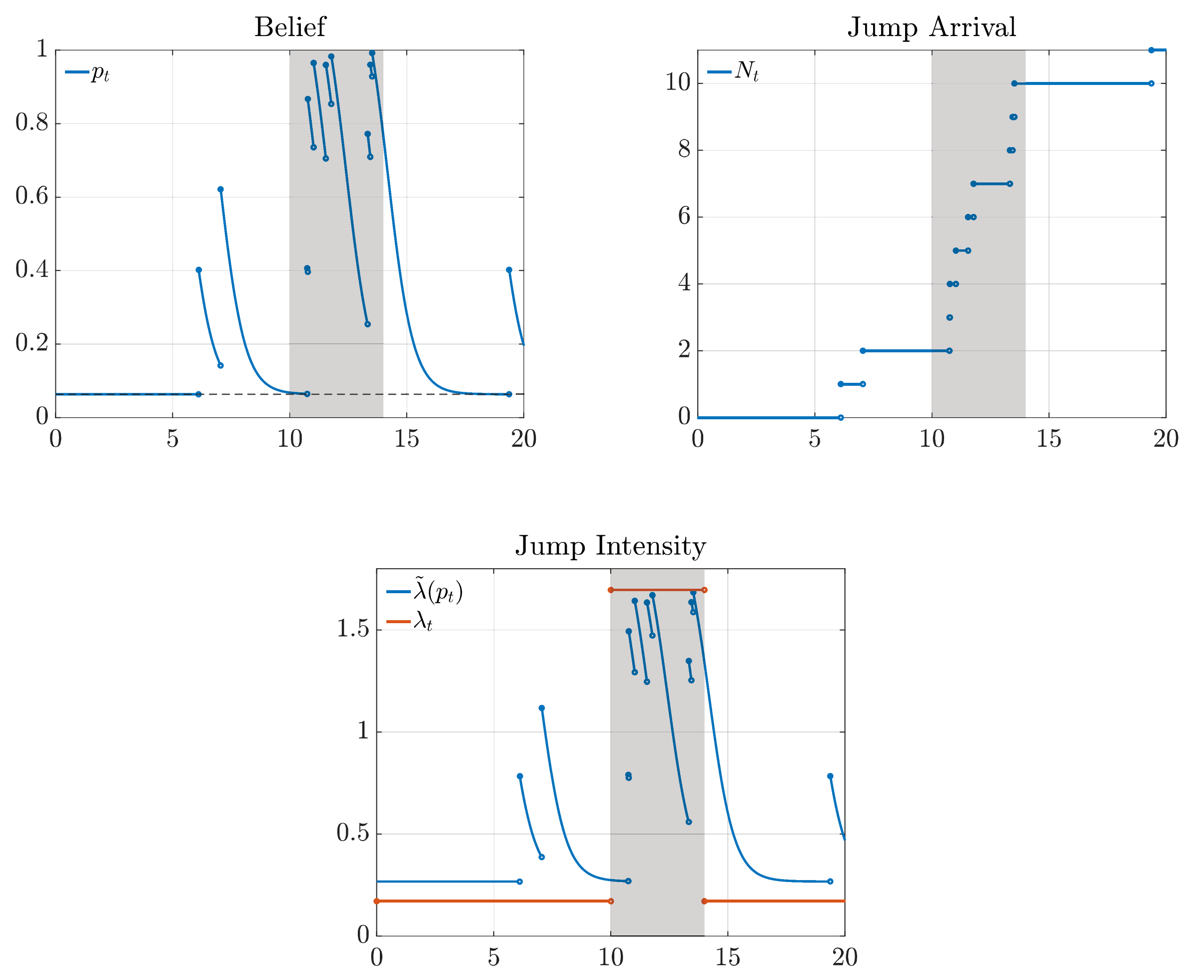

Figure 3: Learning about Risk and Welfare

This figure illustrates the function $V(p_{t-})$. As p_{t-} increases, $V(p_{t-})$ decreases leading to a decrease in the agent's indirect utility, as can be seen from equation (6). The dashed red line represents p^*, the lower bound for p_t. Model parameters are set to the values reported in Tables 1, 3 and 5.

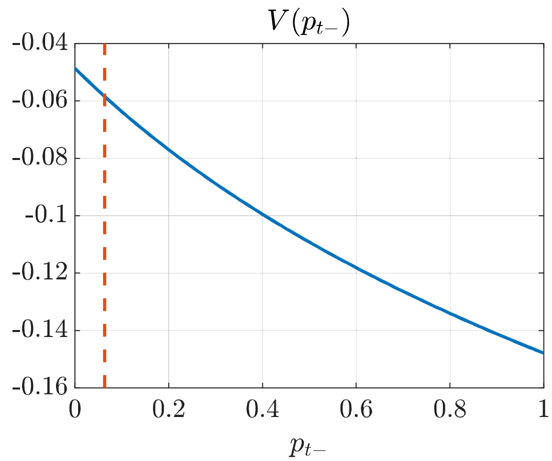

Figure 4: Learning and Amplification

This figure depicts $a(p_{t-})$ representing the signed change in $V(p_{t-})$ due to an upward jump in p_t. For example, if a jump is observed when p_{t-} is 0.2, $V(p_{t-})$ will drop by approximately -0.05. Given the relationship of $V(p_{t-})$ and the agent's indirect utility, see equation (6), this quantity can be interpreted as an additional impact of a downward jump in consumption on welfare via the learning channel equivalent to a further decrease in consumption of approximately 5%. The magnitude of this amplification effect increases in the agent's belief uncertainty and vanishes as the agent's belief approaches perfect certainty. The dashed red line represents p^*, the lower bound for p_t. Model parameters are set to the values reported in Tables 1, 3 and 5.

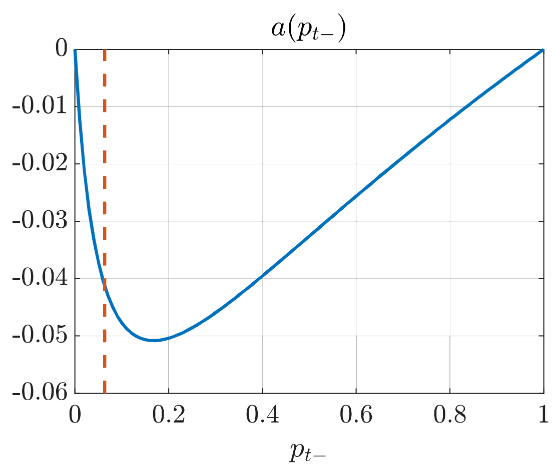

Figure 5: Risk Distortion Factor

This figure plots the risk distortion factor $\omega(p_{t-})$ linking the subjective consumption jump arrival intensities under the physical and the risk-neutral measure, see (11). As is to be expected, for any p_{t-} the jump arrival intensity is higher under $\mathbb{Q}$ given that $\omega(p_{t-}) \geq \frac{1-J_c}{1-J_c(1+\gamma)} > 1$ for all p_{t-}. It decreases as the agent becomes more certain about the current state and increases in belief uncertainty. The dashed red line represents p^*, the lower bound for p_t. Model parameters are set to the values reported in Tables 1, 3 and 5.

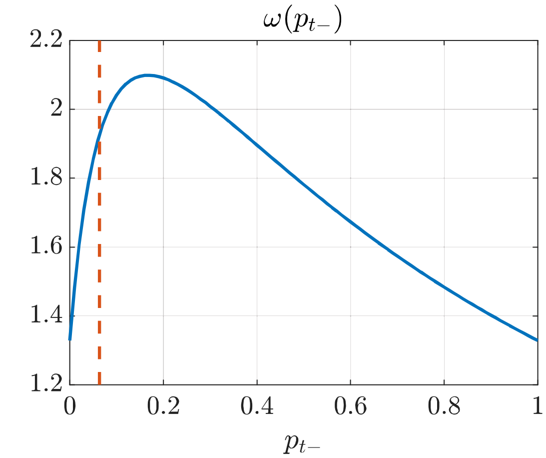

Figure 6: Risk-free Rate

This figure illustrates the locally risk-free rate $r(p_{t-})$. Uncertainty about the state of the world makes $r(p_{t-})$ time-varying. As the agents belief rises, so does the demand for safe assets, driving up the risk-free bond price and depressing the equilibrium risk-free rate which eventually turns negative for $p_{t-} > 0.7$. The dashed line represents p^*, the lower bound for p_t. Model parameters are set to the values reported in Tables 1, 3 and 5.

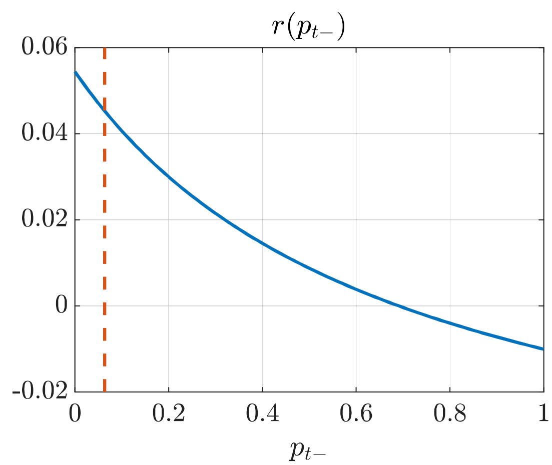

Figure 7: Unlevered Price-Earnings Ratio

This figure displays the unlevered price-earnings ratio $p_X(p_{t-})$. Uncertainty about the state of the world creates fluctuations in firm-level price-earnings ratios. As the agents belief rises, prices and price-earnings ratios of risky assets fall. This is because downward jumps in consumption and perfectly coinciding jumps in earnings are perceived to be more likely when p_{t-} is high, thus making risky assets even riskier. The dashed red line represents p^*, the lower bound for p_t. Model parameters are set to the values reported in Tables 1, 3 and 5.

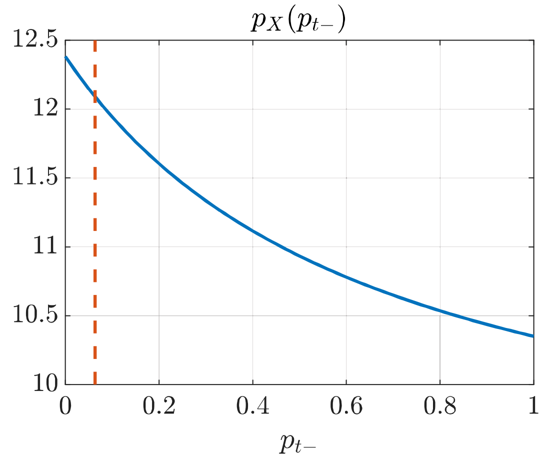

Figure 8: Optimal Default Boundary

This figure shows the optimal default boundary $X_D(p_{t-})$ of a firm that issued debt when its earnings were $X_t = 1$ and the belief about the state of the economy was $p_t = p^*$. Default is declared when X_t falls below $X_D(p_{t-})$ for the first time. Uncertainty about the state of the world makes $X_D(p_{t-})$ time-varying. In expectation, the increased frequency of drops in firm earning in the high risk state makes the firm less profitable and increases the probability that potential losses incurred by equity holders over the short term will not be recouped in the long-run. This makes earlier default, i.e. default at higher levels of X_t, optimal. Therefore, $X_D(p_{t-})$ is increasing in p_{t-}. The dashed red line represents p^*, the lower bound for p_t. Model parameters are set to the values reported in Tables 1, 3 and 5.

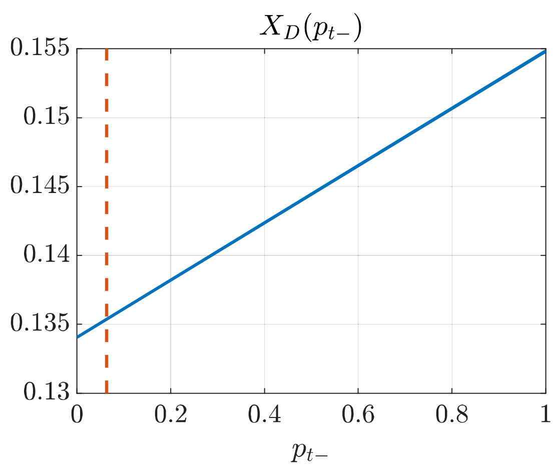

Figure 9: Histogram of Consumption Disasters

This figure compares the distribution of consumption disaster sizes and durations as implied by the model (top panels) and observed in historical data (bottom panels). The empirical distributions of of sizes and durations are constructed from the tables reported in Barro and Ursua (2012). Model parameters are set to the values reported in Table 1. We define disasters, in accordance with the empirical methodology of Barro and Ursua (2012), as path segments during which consecutive annual growth rates of consumption are negative and cumulatively result in a drop of more than 10%. Disaster durations are reported as years of consecutive negative annual growth. Consumption drops are reported as cumulative simple growth rates over the entire duration of a disaster event.

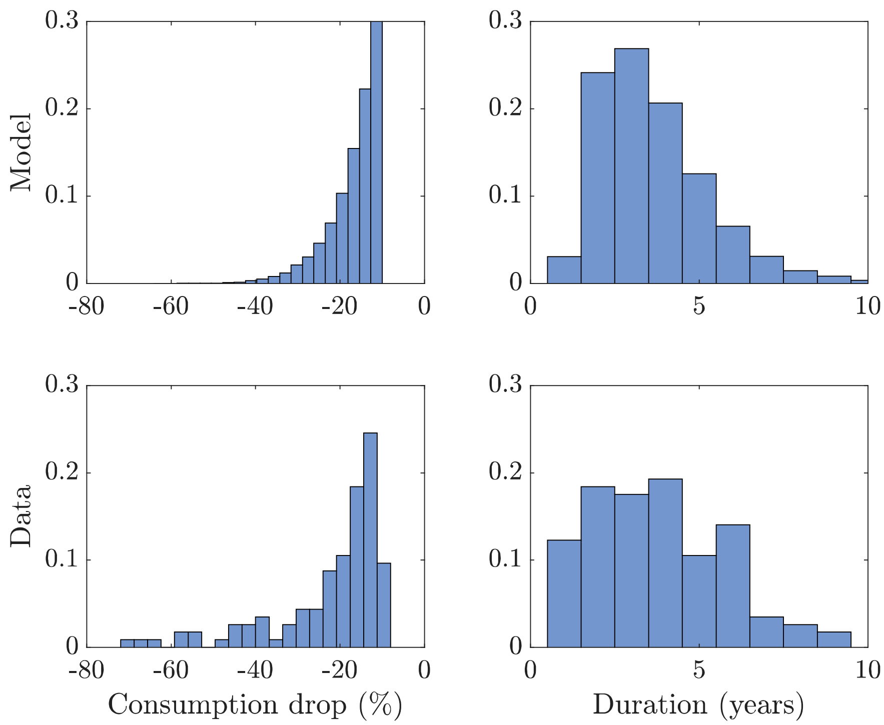

Figure 10: CDX Pricing

This figure depicts the term structure of CDX rates in the left panel and the term structure of physical and risk-neutral default probabilities in the right panel. CDX spreads are annual and reported in basis points per unit of notional for contracts with a fixed time to maturity from 1 to 10 years. Model parameters are set to the values reported in Tables 1, 3 and 5. Empirical averages are computed from daily data on Markits North American Investment Grade CDX Index obtained from ICE Data Services for the period from September 2003 to June 2022. Default probabilities are reported in percent for horizons ranging from 1 to 10 years. Empirical default probabilities are the average cumulative issuer-weighted global default rates reported by Moodys spanning the period from 1920 to 2017 for entities categorized as investment grade (letter rating of Baa3 or better).

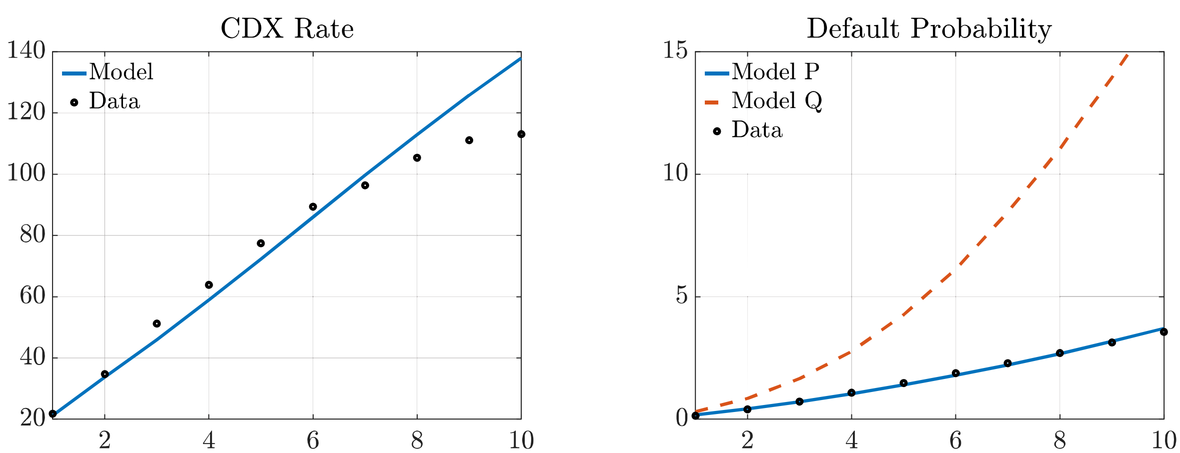

Figure 11: Equity Risk Premium

This figure displays the conditional equity risk premium of unlevered firms (left panel) and levered firms (right panel). The blue lines represent the learning model. The yellow lines represent the model with full information in the high-risk state (yellow markers) and low-risk state (black markers). For the levered firms, earnings are normalized to 1. The initial belief for the learning model is set to p^* and the inital state for the model with full information is set to λ_L. The dashed line represents p^*, which is the lower bound for p_t. The remaining model parameters are set to the values reported in Table 1.

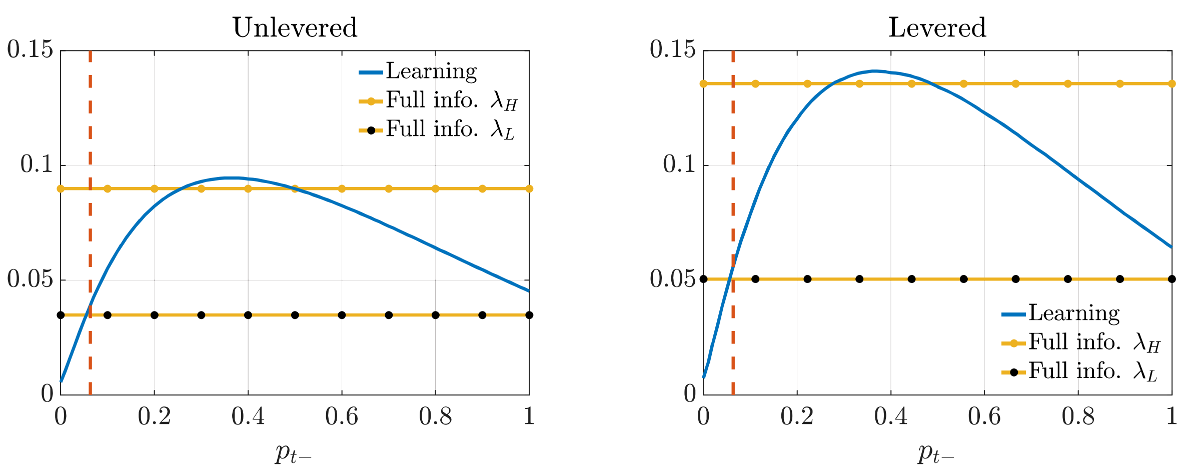

Figure 12: Time Series of CDX and Leverage

This figure displays the empirical time series of the 5-year maturity spreads for the Markit North American Investment Grade CDX Index and its model-implied equivalent in basis points (top panel), alongside the data on the cross-sectional average CDX leverage in percent (bottom panel), covering the period from September 2003 to June 2022. The average CDX leverage is calculated using available CRSP-Compustat data on book debt and market equity, by referencing the constituent list for each CDX series. To generate the time series for the model-implied CDX, we utilize the leverage time series from the bottom panel and set the unobserved belief to closely align with the empirical time series of CDX spreads. Model parameters are set to the values reported in Tables 1, 3 and 5.

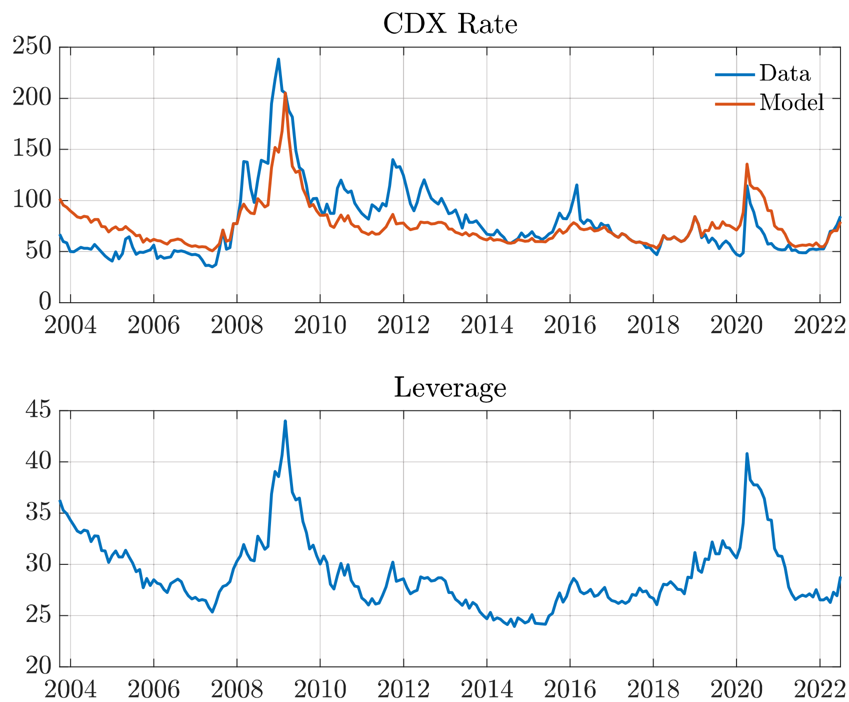

Figure 13: Great Depression – Consumption

This figure presents cross-sectional averages of 10,000 model simulations, where the cumulative drop in annual consumption growth equals approximately 17%, akin to the decline observed during the Great Depression. The simulated data is aligned so that the beginning of a crisis corresponds to time 0, which is the beginning of 1930 in the data. Model parameters are set to the values reported in Table 1. The first panel shows the cross-simulation average of annual cumulative consumption growth from the model and the actual data observed during the Great Depression. Both model and actual data are normalized such that cumulative consumption growth equals one before the first drop in annual consumption occurs. The second panel displays the average number of jump arrivals in consumption and earnings. The blue bars represents the average number of jumps observed during a particular year, and the red line represents the average cumulative number of jumps, starting at the onset of the observed disaster. The third panel displays the average simulated daily time series of beliefs and the average actual state of the Markov chain.

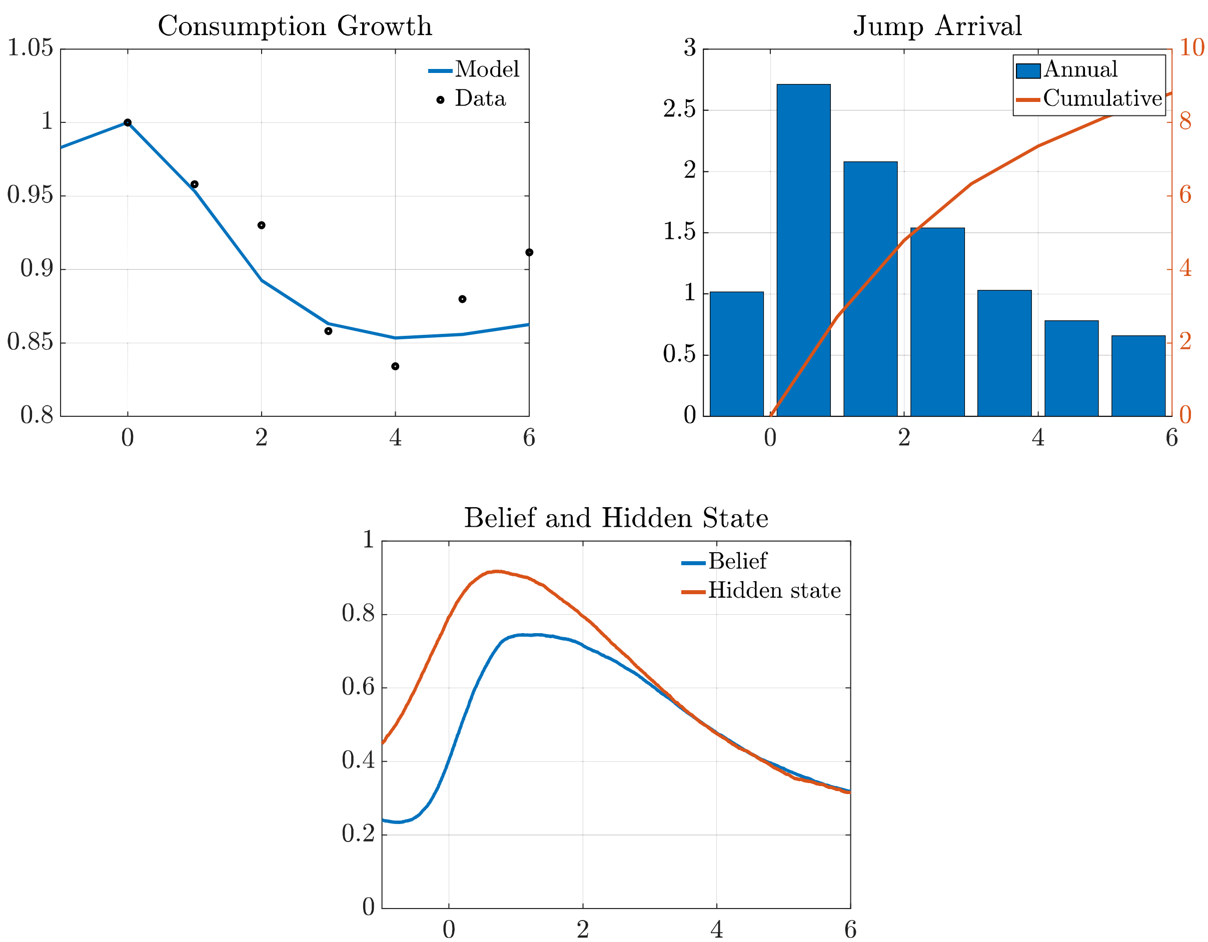

Figure 14: Great Depression – Credit Market

This figure presents cross-sectional averages of 10,000 model simulations, where the cumulative drop in annual consumption growth equals approximately 17%, akin to the decline observed during the Great Depression. The simulated data is aligned so that the beginning of a crisis corresponds to time 0, which is the beginning of 1930 in the data. The results are averaged across 125 individual firms and simulations. Model parameters are set to the values reported in Table 1. In the top panels, the figure depicts the average 5-year CDX rate in basis points (left panel), and average leverage in percent (right panel). Blue lines represent the learning model, red lines represent the model with full information. In the bottom panels, the figure depicts average physical and risk-neutral 5-year default probabilities in percent (using blue lines and red lines respectively) for the learning model (left panel) and model with full information (right panel).

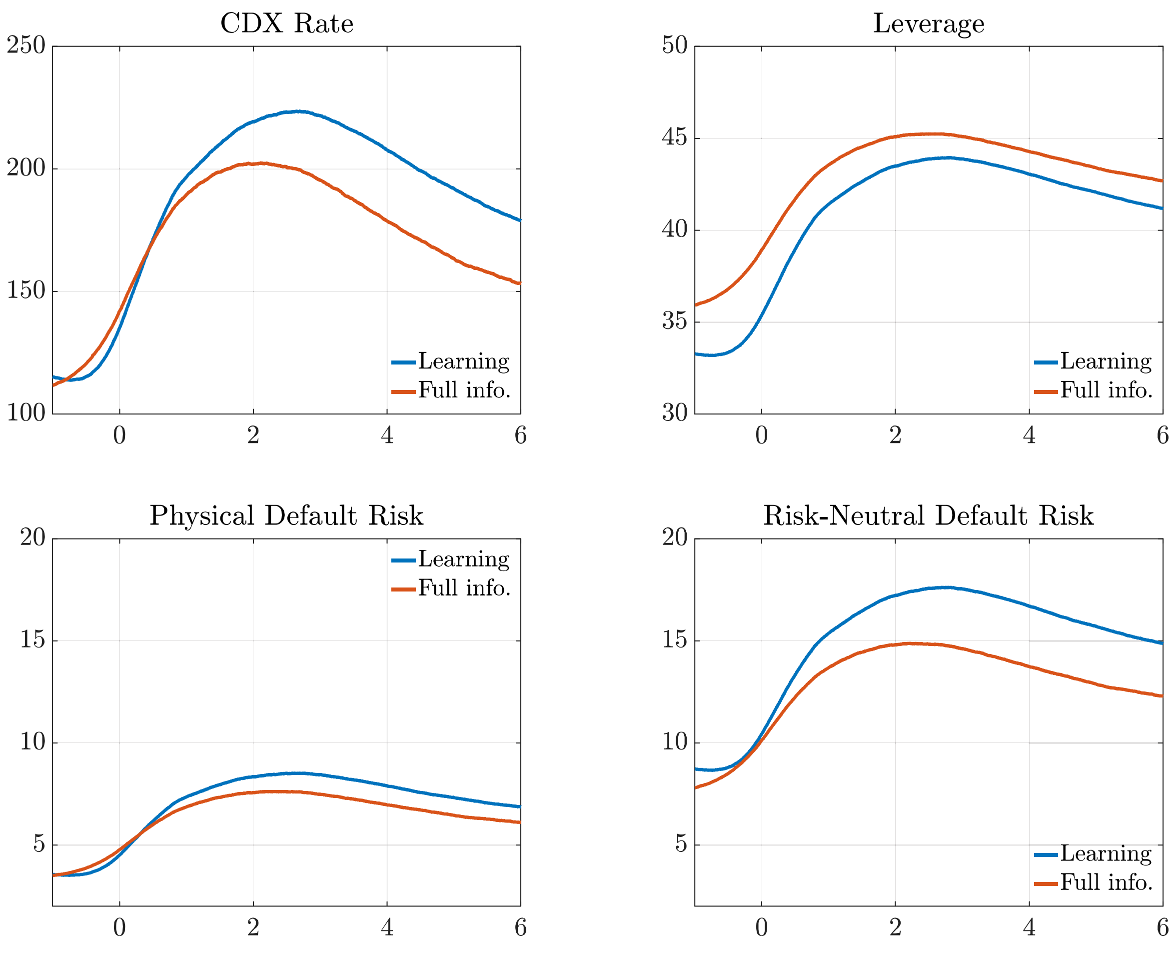

Figure 15: Great Depression – Equity Market

This figure presents cross-sectional averages of 10,000 model simulations, where the cumulative drop in annual consumption growth equals approximately 17%, akin to the decline observed during the Great Depression. The simulated data is aligned so that the beginning of a crisis corresponds to time 0, which is the beginning of 1930 in the data. The red line represents the daily, cum-dividend, inflation-adjusted cumulative equity return on the CRSP Index observed during the Great Depression. The blue line represents the daily, cum-dividend, cumulative equity return generated by a model, averaged across 125 individual firms and simulations. The shaded areas represent the 50, 80 and 90 percent confidence intervals across simulations. The figure depicts the results for the model with full information without leverage (top left panel), the model with full information and leverage (top right panel), the learning model without leverage (bottom left panel), the model with learning and leverage (bottom right panel).

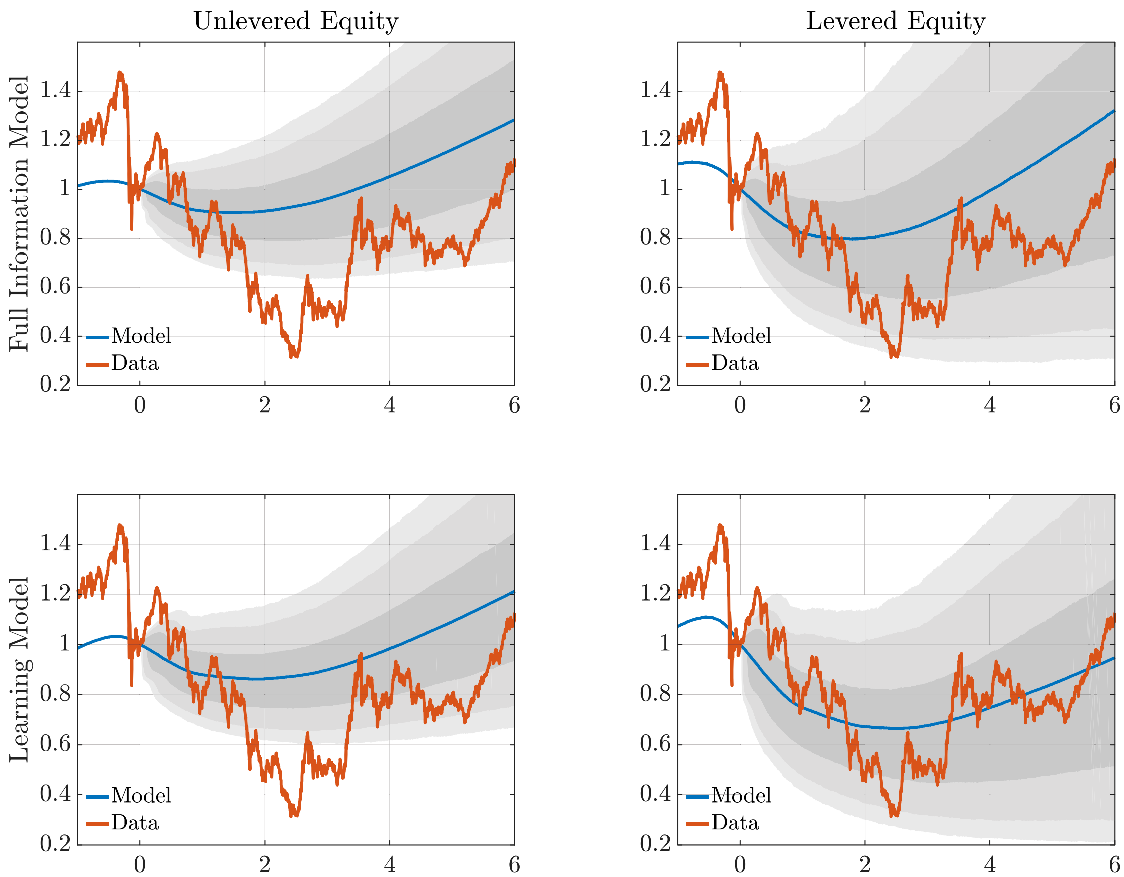

Figure 16: Great Depression – Equity Risk Premium

This figure presents cross-sectional averages of 10,000 model simulations, where the cumulative drop in annual consumption growth equals approximately 17%, akin to the decline observed during the Great Depression. The simulated data is aligned so that the beginning of a crisis corresponds to time 0, which is the beginning of 1930 in the data. For the case of levered equity the results are averaged across 125 levered individual firms in addition. Model parameters are set to the values reported in 1. The figure depicts the daily time series of both the unlevered (red line, right y-axis) and the levered (blue line, left y-axis) conditional equity risk premiums. The graphs are vertically aligned such that they begin at the same point and scaled such that their local maximum and inflection point are at the same height.

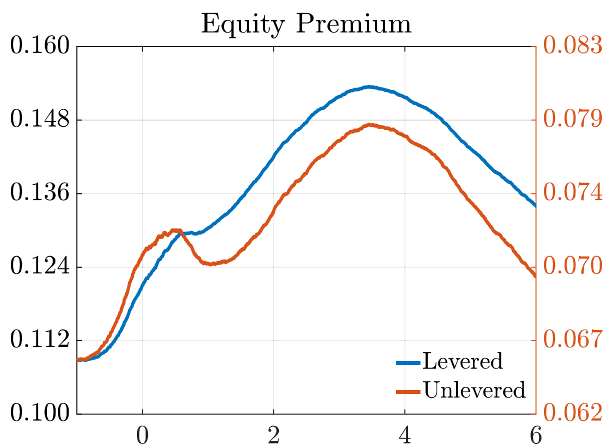

Table 1: Model Parameters

This table summarizes all the exogenous parameters for the model. There are 17 parameters in total: 7 for the consumption process, 2 for preferences, 5 for the firm-level earnings process, and 3 for corporate financing decisions.

Panel A: Consumption Process

Consumption growth rate	μ_c
Consumption growth volatility	σ_c
Jump intensity state L	λ_L
Jump intensity state H	λ_H
Markov chain transition intensity from state L to H	ϕ_{LH}
Markov chain transition intensity from state H to L	ϕ_{HL}
Jump size	$1/\epsilon_c$

Panel B: Preferences

Time discount rate	β
Risk aversion	γ
Elasticity of intertemporal substitution	ψ

Panel C: Firm-level Earning Process

Consumption-earnings correlation	ρ_{cx}
Earnings growth rate	μ_x
Idiosyncratic risk	σ_x^{id}
Systematic risk	σ_x^{sys}
Exogenous exit rate	λ_δ

Panel D: Corporate Financing Parameters

Corporate tax rate	η
Debt issuance costs	ι
Bankruptcy recovery fraction	α

Table 2: Consumption Dynamics

Panel A: Parameters

Consumption growth rate	μ_c	0.0240
Consumption growth volatility	σ_c	0.0113
Jump intensity state L	λ_L	0.1708
Jump intensity state H	λ_H	1.6960
Markov chain transition intensity from state L to H	ϕ_{LH}	0.1224
Markov chain transition intensity from state H to L	ϕ_{HL}	0.3816
Jump size	$1/\epsilon_c$	0.0247

Panel B: Long Sample

	Data	Model
Mean disaster size	0.1659	0.1661
Mean disaster duration	3.6667	3.6567
Likelihood of disasters	0.0360	0.0360

Panel C: Post-War Sample

	Data	Model
Mean consumption growth	0.0183	0.0183
Std. dev. of consumption growth	0.0154	0.0151
Skewness of consumption growth	-1.1000	-1.1017
Kurtosis of consumption growth	5.7480	4.9636

Table 3: Predefined Parameters

Parameter		Value
Time discount rate	β	0.0435
Risk aversion	γ	10
Elasticity of intertemporal substitution	ψ	1
Consumption-earnings correlation	ρ_{cx}	0.2
Earnings growth rate	μ_x	0.0511
Exogenous exit rate	λ_δ	0.011
Corporate tax rate	η	0.154
Debt issuance costs	ι	0.01

Table 4: Sensitivity Matrix

This table shows the sensitivity of model-implied moments (in rows) with respect to model parameters (in columns). The sensitivity of moment i with respect to parameter j equals $\frac{\partial \Psi_i^M}{\partial \theta_j} \frac{\theta_j}{\Psi_i^M}$ and is evaluated at the vector of point estimates from Table 5.

	σ_x^{id}	σ_x^{sys}	$1-\alpha$	φ
Average excess return	0.31	-0.05	-0.39	1.13
Average leverage	0.75	-0.19	-1.04	0.11
Average 5-year CDX rate	1.87	-0.12	-1.26	2.04
Std. dev. of excess returns	1.30	-0.01	-0.61	0.36
Std. dev. of market excess returns	0.40	0.21	-0.38	0.79
Std. dev. of leverage	0.70	-0.02	-0.39	0.10
Std. dev. of 5-year CDX rate	1.27	-0.11	-1.55	2.64

Table 5: Estimated Parameters

Parameter		Value
Idiosyncratic risk	σ_x^{id}	0.1809
		(0.0073)
Systematic risk	σ_x^{sys}	0.0522
		(0.0437)
Bankruptcy costs	$1-\alpha$	0.3435
		(0.0841)
Jump scaling parameter	φ	3.2787
		(0.3282)

Table 6: SMM Moments

Moments	Data	Model
Average excess return	0.0090	0.0079
Average leverage	0.2894	0.2737
Average 5-year CDX rate	0.0077	0.0072
Std. dev. of excess returns	0.0917	0.0977
Std. dev. of market excess returns	0.0505	0.0418
Std. dev. of leverage	0.1589	0.1533
Std. dev. of 5-year CDX rate	0.0034	0.0038

Table 7: CDX Moments

	CDX Rate		$\mathbb{P}$ Def. Prob.		$\mathbb{Q}$ Def. Prob.
Horizon	Data	Model	Data	Model	Model
1-year	0.0022	0.0021	0.0014	0.0018	0.0031
2-year	0.0035	0.0034	0.0040	0.0042	0.0085
3-year	0.0051	0.0046	0.0072	0.0071	0.0166
4-year	0.0064	0.0059	0.0108	0.0103	0.0277
5-year	0.0077	0.0072	0.0148	0.0140	0.0427
6-year	0.0089	0.0086	0.0188	0.0179	0.0612
7-year	0.0096	0.0100	0.0229	0.0221	0.0845
8-year	0.0105	0.0113	0.0270	0.0267	0.1104
9-year	0.0111	0.0126	0.0313	0.0318	0.1394
10-year	0.0113	0.0138	0.0356	0.0371	0.1707