Subjective Risk and Return

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Abstract

Traditional asset pricing models like the CAPM explain realized returns worse than newer asset pricing models like Fama-French-5, but why? I show that traditional models explain subjective risk and return expectations well but also predict return-subtracting mispricing. Newer models, by contrast, explain subjective risk and return expectations poorly but predict return-enhancing mispricing. These results imply that the CAPM is a good model of risk but fails to explain realized returns because risk is correlated with mispricing. I explain this disconnect by a model in which all the CAPM assumptions hold, except that some investors have an optimism bias.

Keywords: Asset pricing, asset pricing models, equity factors, subjective risk, mispricing, optimism bias

JEL Codes: G4, G11, G12, G14

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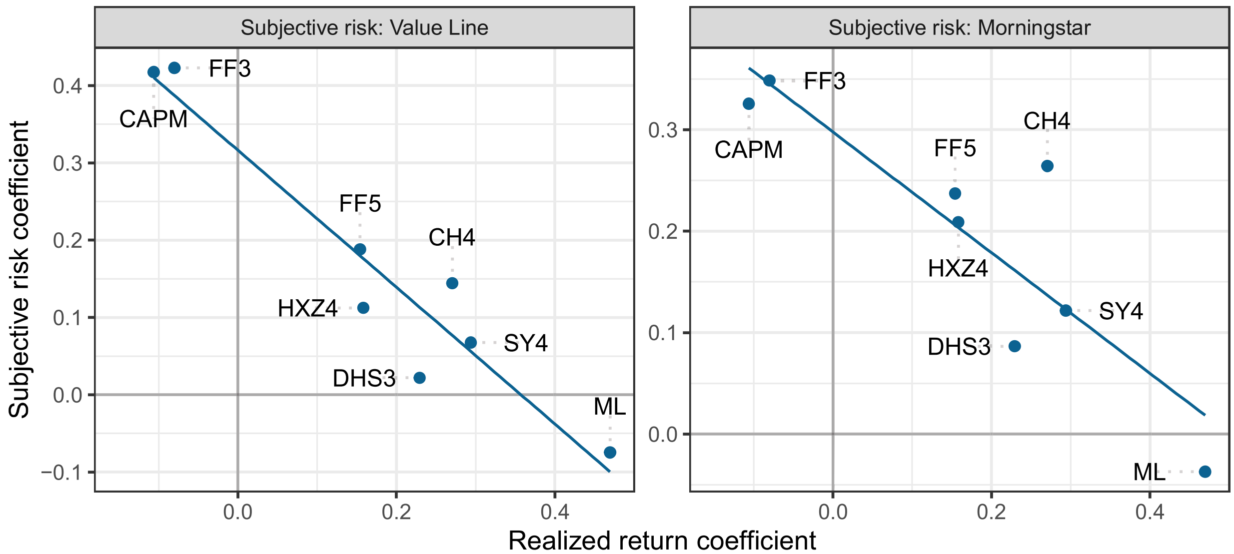
Theoretical asset pricing models like the CAPM rest on a solid theoretical foundation but cannot explain average realized stock returns. Empirical asset pricing models like the Fama and French (2015) model, by contrast, have no theoretical foundation but explain realized returns well. But why does the CAPM fail while the newer empirical asset pricing models succeed? The rational interpretation is that the newer models better capture how investors perceive risk (Fama and French, 1993), while the behavioral interpretation is that the newer models better capture mispricing (Bordalo et al., 2024). Whether the rational or the behavioral interpretation is correct determines whether the stock market is efficient or inefficient, but the models' ability to explain realized returns cannot be used to make the distinction (Kozak et al., 2018). Instead, I propose to interpret asset pricing models by ranking them on their ability to explain observable proxies for risk and mispricing.

Asset pricing models and realized returns. I consider seven factor-based models and one (ML) based on a machine learning model trained to predict realized returns. The factor-based models are usually applied to explain portfolio returns, but the risk and mispricing proxies are available at the individual stock level. To handle this discrepancy, I propose a method—inspired by the stochastic discount factor (SDF) framework—to compute the model-implied risk of a stock. For factor-based models, the model-implied risk is the conditional covariance between a stock's return and the return of the model-implied SDF. For the ML model, the model-implied risk is simply the stock's predicted return.

While the approach is non-standard, it results in the expected ranking of models in terms of their ability to explain realized returns. To substantiate this claim, the x-coordinate in all panels of Figure 1 shows the slope coefficient from regressing a stock's realized return in month $t + 1$ on its model-implied risk at time t. The model-implied risk is standardized to have a mean of zero and a variance of one for all models, so a higher slope coefficient translates into a higher R^2. Of the eight models I consider, the CAPM is the worst at explaining realized returns, newer factor-based models like Fama and French (2015) and Hou et al. (2015) are better, and the ML model is the best.

(a) Models that explain realized returns well explain subjective risk poorly



(b) but align with return-enhancing mispricing

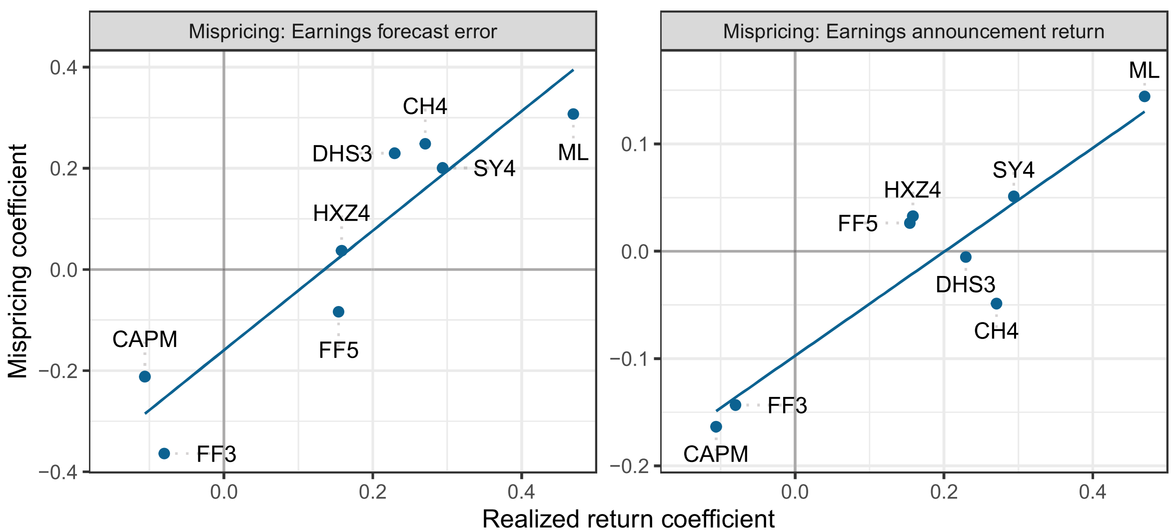


Figure 1: Do asset pricing models capture risk or mispricing?

Note: The figure summarizes the main empirical findings of the paper. For each of the eight asset pricing models described in Table 1, I compute their model-implied risk prediction at the individual stock level as explained in Section 1.2. The x-coordinate in the figure is the β^{x} coefficient from: $r_{i,t+1} = \alpha_t + \beta^{\mathrm{x}}\mathrm{mrisk}_{i,t} + \epsilon_{i,t}$, where $r_{i,t+1}$ is the next-month realized excess return and α_t is a time-fixed effect, and $\mathrm{mrisk}_{i,t}$ is the model-implied risk. In panel (a), the y-coordinate is the β^{risk} coeffiecient from: $\mathrm{srisk}_{i,t} = \alpha_t + \beta^{\mathrm{risk}}\mathrm{mrisk}_{i,t} + \epsilon_{i,t}$, where $\mathrm{srisk}_{i,t}$ is a subjective risk rating from Value Line or Morningstar. In panel (b), the y-coordinate is the β^{mp} coefficient from: $\mathrm{mp}_{i,t} = \alpha_t + \beta^{\mathrm{mp}}\mathrm{mrisk}_{i,t} + \epsilon_{i,t}$, where $\mathrm{mp}_{i,t}$ the earnings forecast error (the next fiscal year realized earnings minus the time t consensus forecast from I/B/E/S) or the market-adjusted earnings announcement day return. The solid blue line shows the best linear fit.

Asset pricing models and risk. The first proxy for risk is based on subjective risk ratings from equity analysts. Analysts sell their advice to investors, so they have an incentive to provide risk ratings that align with how investors perceive risk. The data comes from two independent equity research firms: Value Line and Morningstar. Value Line provides a safety rank that depends on a qualitative and quantitative assessment of the stock's price stability and the financial strength of the underlying firm. Morningstar provides a cost of equity, which depends on a qualitative assessment of the stock's systematic risk. The qualitative aspect of the subjective risk ratings ensures that they provide a model-independent measure of which stocks analysts (and, by extension, the investors they serve) perceive as risky.

Figure 1 panel (a) shows that models that explain realized returns well explain subjective risk poorly. Specifically, the y-coordinate shows the slope coefficient from regressing the subjective risk rating on the model-implied risk. The best model of subjective risk from Value Line is the CAPM closely followed by the Fama and French (1993, FF3) model, whereas the best model of subjective risk from Morningstar is FF3 closely followed by the CAPM. Newer factor-based models like FF5 and HXZ4 are much worse at explaining subjective risk, and ML is the worst model overall.

The second proxy for risk is subjective return expectations from Value Line, Morningstar, and I/B/E/S. Subjective expected returns from the perspective of the representative investor only depend on risk, so the idea is to use analyst expectations as a proxy.[1] The results are similar to those using subjective risk. The CAPM and FF3 are the best at explaining subjective expected returns, while newer models, especially ML, are the worst. For example, the CAPM and FF3 have a significantly positive relationship to subjective expected returns from I/B/E/S, while all other models have a significantly negative relationship. Overall, the fact that the CAPM and FF3 are the best at explaining subjective risk and subjective

[1] Subjective expected returns are a less direct proxy than subjective risk ratings because they depend both on perceived risk and perceived mispricing, that is, whether the analyst thinks the stocks are over- or undervalued relative to the representative investor.

expected returns is direct evidence against the rational interpretation that newer models better capture risk.

Asset pricing models and mispricing. If newer models do not capture risk they must capture mispricing. To test this idea directly, I rely on two different mispricing proxies. The first proxy is earnings forecast errors, computed as the difference between the realized next-year earnings and the current consensus expectation from I/B/E/S (scaled by the stock's price). The left plot in Figure 1 panel (a) is similar to panel (a) except that the y-coordinate is the slope coefficient from regressing earnings forecast errors on model-implied risk. The figure shows that models that explain realized returns well also tend to predict positive earnings forecast errors (positive earnings surprises). By contrast, the CAPM and FF3 tend to predict negative earnings forecast errors. The results are robust across forecast horizons from one-quarter ahead to 5-years ahead. The results also do not appear to reflect sell-side analysts' incentives-related biases as the results are similar when using earnings forecasts provided by the Value Line.

My second proxy for mispricing is the returns on earnings announcement days. Earnings announcements are periods in which a lot of information is revealed and should, therefore, be a time when biases in earnings expectations are partly corrected. The right plot of Figure 1 panel (b) is similar to the left plot, except that the y-coordinate is the slope coefficient from regressing the market-adjusted return of a stock on the day of its earnings announcement on its model-implied risk. The results are similar to those using earnings forecast errors. The CAPM and FF3 tend to predict negative earnings announcement returns, while newer models, especially ML, tend to predict positive earnings announcement returns. Overall, the results for earnings forecast errors and earnings announcement returns provide strong evidence in favor of the behavioral interpretation that newer models better capture mispricing.

Interpreting equity factors. Most asset pricing models consist of equity factors, so in the last empirical part of the paper, I test whether 119 equity factors are best interpreted as capturing behavioral mispricing or rational compensation for risk. Specifically, I classify the factors into four groups based on the correlation between the underlying factor characteristic and the two subjective risk ratings (the risk coordinate) and its correlation to earnings forecast errors and earnings announcement returns (the mispricing coordinate).

The largest group, with a share of 53%, is mispricing factors, which have a negative correlation with risk (inconsistent with the rational interpretation) but a positive correlation with mispricing (consistent with the behavioral interpretation). The second largest group, with a share of 22%, are risk factors that have a positive correlation with risk and a negative correlation with mispricing. The third largest group (share of 20%) is mispricing-risk factors that have a positive correlation to both risk and mispricing, and the smallest group (share of 4%) is anomalies that have a negative correlation to both risk and mispricing. The results suggest that in the zoo of equity factors most reflect mispricing but some reflect risk.

The Optimism-Adjusted CAPM. The results in Figure 1 suggest that the CAPM is a good model of risk but fail to explain realized returns because investors are too optimistic about the cash flow of high beta stocks. These results can not be explained by models that modify the CAPM by introduction frictions, such as leverage constraints (Black et al., 1972; Frazzini and Pedersen, 2014), or short-selling constraints (Miller, 1977; Hong and Sraer, 2016), because the average investors in these models have unbiased expectations. Therefore, I propose a new model that modifies an otherwise standard CAPM setup with the assumption that some investors have an optimism bias.[2]

I start by modeling a single investor trying to forecast a stock's dividend. The investor receives two independent signals whose dispersion is proportional to the stock-specific dividend variance. The correct (Bayesian) approach is to equal-weight the two signals, but I incorporate optimism bias by assuming that the investors put more weight on the highest signal. Proposition 1 shows that the bias of an optimistic investor increase in the stock's

[2]Optimism bias has been widely documented in psychology and neuroscience. For example, Sharot (2011) reports that a robust estimate across a range of forecasting tasks is that 80% of the population issue excessively optimistic forecasts, and Kahneman (2011, p. 255) argues that optimism bias could be the most significant cognitive bias in terms of its consequences for decision-making.

dividend variance. Intuitively, stocks with more uncertain dividends leaves more room for optimism. Empirically, cash flow uncertainty is strongly increasing in beta, so Proposition 1 can explain why high beta stocks tend to have too optimistic cash flow expectations.

To study prices and returns, I model an economy with multiple investors that receive the same two signals for each stock but differ in their relative optimism (how much weight they put on the highest signal). The optimism of the representative investor depends on the relative wealth controlled by optimists and pessimists and is attenuated by the wealth controlled by investors with rational beliefs. I assume that all investors have mean-variance preferences, which means that, from the perspective of the representative investor, the risk of a stock is determined by its market beta, and subjective expected return follows the CAPM. Proposition 2, however, shows that objective expected returns follow the Optimism-Adjusted CAPM, in that they increase in market beta but decrease in return volatility. Empirically, stocks with high market beta tend to have high return volatility, so the model predicts that the relationship between realized return and market beta is weaker than the representative investor expects. Proposition 2 can, therefore, explain how the CAPM can be a bad model of average realized returns even though it is a good model of risk.

Related literature. Few papers study subjective risk ratings. A notable exception is Lui et al. (2007) who regres subjective risk ratings from Salomon Smith Barney (now Citigroup), Merril Lynch, and Value Line on various security characteristics. They find that subjective risk tends to increase in a stock's market beta and book-to-market equity and decrease in its size—consistent with the CAPM and FF3. Relatedly, Gormsen and Huber (2023) finds that CFOs' cost of capital estimates are directionally consistent with the CAPM and FF3.[3]

[3]Several papers test the relationship between stock characteristics and subjective expected returns. For example, Brav et al. (2005) show that Value Line's expected returns are directionally consistent with the CAPM and FF3, but not with CH4, and Bastianello (2022) show that I/B/E/S expected returns are directionally consistent with the CAPM and FF3. Subjective expected returns, however, mainly reflect the analyst's view on whether the stock is mispriced, not whether it is risky. For example, the average correlation between Morningstar's risk ratings and expected returns is only 0.09. This low correlation suggests that an

There are comparatively more papers that study earnings forecast errors and earnings announcement returns as proxies for mispricing. For example, Bordalo et al. (2024) shows that biased earnings expectations from I/B/E/S can explain most of the realized returns of the size, value, momentum, investment, and profitability factors, and Engelberg et al. (2018) shows that most equity factors studied in the literature have higher returns on earnings announcement days and predict positive earnings forecast errors.

My first contribution to this literature is that I am the first (to the best of my knowledge) to study asset pricing models' ability to explain risk and mispricing proxies jointly. By studying both aspects jointly, I can show that the reason why market beta explains subjective risk ratings and CFO cost of equity estimates well but realized returns poorly is that it also predicts return-subtracting mispricing. I can also show that most equity factors *only* reflect mispricing as they are negatively correlated with risk. My second contribution is that I study the full implication of an asset pricing model rather than considering each factor in isolation. For example, existing papers might evaluate FF3 by testing whether the beta, size, and value characteristics (or the corresponding factor) are significantly related to risk or mispricing, but these factors are not equally important for FF3 (for example, the SDF weight of the value factors is four times that of the size factor), which makes it difficult to evaluate the implication for the full model. My methodology allows me to provide the first (to the best of my knowledge) ranking of leading asset pricing models in terms of their ability to explain risk and mispricing. My results imply that the search for better models of realized returns has resulted in better models of mispricing but worse models of risk.

asset pricing model's ability to explain subjective expected returns could differ greatly from its ability to explain subjective risk.

1 Data and Empirical Methodology

This section explains the subjective risk ratings from Value Line and Morningstar, as well as the risk measure from seven factor-based asset pricing models and one model based on machine learning. Table 1 provides an overview of the risk measures. The remaining data are described in Appendix A.2.

Table 1: Risk measures

Name	Abbreviation	From	Type	Start
Safety rank		Value Line	Subjective risk	1990
Cost of equity		Morningstar	Subjective risk	2003
CAPM	CAPM	Sharpe (1964)	Model risk	1972
Fama-French-3	FF3	Fama and French (1993)	Model risk	1972
Fama-French-5	FF5	Fama and French (2015)	Model risk	1972
Investment CAPM	HXZ	Hou et al. (2015)	Model risk	1972
Mispricing Factors	SY	Stambaugh and Yuan (2017)	Model risk	1972
Behavioral Factors	DHS	Daniel et al. (2020)	Model risk	1972
Machine Learning	ML	Gu et al. (2020)	Model risk	1972

Note: The table shows the two subjective risk measures and the equity research firm they come from. The table also shows the eight model-based risk measures, a name abbreviation, and a paper reference. The *start* column shows the first date a given risk measure is available and the last date for all measures is December 2021.

1.1 Subjective risk

1.1.1 The Safety rank from Value Line

The first subjective risk measure comes from Value Line, an independent research firm founded in 1931 that currently employs 70+ equity analysts. Value Line's customers range from individual investors who pay an annual subscription fee of $795 for basic services to professional investors who pay more than $100,000 annually.[4] Prominent investors who have

[4] Value Line's annual report from 2021, p. 15: "Value Line serves primarily individual and professional investors in stocks, who pay mostly on annual subscription plans, for basic services or as much as $100,000 or more annually for comprehensive premium quality research, not obtainable elsewhere."

used Value Line include Warren Buffett and Charlie Munger (CNBC, 1998) as well as Peter Lynch (Lynch and Rothchild, 2000). Value Line also received considerable academic attention in the 1970s and 1980s for the quality of their stock recommendations (the "timeliness rank"). For example, Black (1973) showed that Value Line had statistically significant stock-picking skills, which he argued provided hope for active managers.

Value Line's flagship product is the weekly publication of the "Value Line Investment Survey," which contains summary statistics for 1,700 of the largest U.S. stocks and an in-depth analysis of 130-140 of these stocks. Each stock gets an in-depth review once per quarter or if something material happens to the underlying company. Appendix A.2.5 shows a Value Line report on Apple.

The primary risk measure from Value Line, and my first measure of subjective risk, is the so-called "safety rank." The safety rank ranges from 1 for the safest stocks to 5 for the riskiest, and is derived by taking an average of a stock's rank with respect to two sub-ratings, namely a "price stability index" and a "financial strength rating."[5] To avoid losing information from putting stocks into discrete bins, I re-define the safety rank as the average cross-sectional rank of price stability and financial strength. In addition, I standardize the average cross-sectional rank by subtracting the cross-sectional mean and dividing it by the cross-sectional standard deviation. The average cross-sectional correlation between the binned version of this transformed measure and the original safety rank is 92%.

I obtain the data directly from Value Line's historical archives and merge it with CRSP (Appendix A.2.4 describes the merging procedure). I restrict the sample to ordinary common stocks (CRSP `shrcd` 10, 11, or 12) traded on NYSE, Amex, and NASDAQ (CRSP `exchcd`

[5]In their database manual, Value Line explains the safety rank as: "Safety is a measurement of the total risk of a stock. Total risk is different from Beta. The latter measures only the extent to which a stock normally responds to changes in the trend and level of the market as a whole. The Safety Rank is a more comprehensive measure of risk, including all those factors peculiar to the company's business, such as its financial condition, management competence, etc. The Safety Rank is derived by averaging two variables: 1) the stock's index of Price Stability, and 2) the Financial Strength rating of the company" (ValueLine, 2021).

1, 2, or 3). The final sample contains observations for a minimum, median, and maximum of 1,080, 1,461, and 1,545 stocks, respectively, on a monthly frequency from July 1990 to December 2021.

1.1.2 The Cost of Equity from Morningstar

The second subjective risk measure comes from Morningstar, an independent research firm founded in 1984. Morningstar is most known for its mutual fund research, which started in 1985. The key output from this research is a "star rating" for each fund in their coverage universe. Ben-David et al. (2022) show that the star rating is the most important determinant of flows into mutual funds, which confirms the widespread adoption of Morningstar's mutual fund research.

In 2001, Morningstar also began to provide research on individual stocks, where the key output is a stock-specific star rating. The main determinant of this star rating is the difference between the stock's price and a fair value calculated by Morningstar. This fair value is derived via a discounted cash flow methodology, where an important input is the stock's cost of equity. Morningstar writes that the cost of equity is the sum of a risk-free rate, common to all stocks, and a stock-specific risk premium, which *depends on the systematic risk of a stock as determined qualitatively by the analyst* (Morningstar, 2022, p. 4).[6] Importantly, this procedure implies that cross-sectional variation in the cost of equity is purely a function of the stock's systematic risk as perceived by Morningstar.[7] Similar to the safety rank, I standardize the cost of equity by subtracting the cross-sectional mean and dividing by the cross-sectional standard deviation.

[6] In practice, Morningstar finds the risk premium by assigning stocks into different systematic risk categories. Therefore, the cost of equity is effectively a discrete variable, which takes between six to nine different values within a specific month.

[7] Morningstar writes that their approach is inspired by the logic of the CAPM, but differs in that they take a qualitative and forward-looking approach. As a result, Morningstar's cost of equity estimate is not simply the result of plugging their market beta estimate into the CAPM, but rather a qualitative assessment of the stock's systematic risk.

I obtain the cost of equity estimate from Morningstar Direct and merge it to the CRSP database (Appendix A.2.4 describes the merging procedure). I restrict the sample to ordinary common stocks listed on NYSE, Amex, or NASDAQ, which results in a minimum, median, and maximum of 235, 658, and 1,137 stocks, respectively, on a monthly frequency from August 2003 to December 2021.

1.2 Model risk

1.2.1 Factor risk

I consider seven factor-based asset pricing models, namely, the CAPM of Sharpe (1964), Lintner (1965), and Mossin (1966), the 3-factor model of Fama and French (1993), the four-factor model of Carhart (1997), the five-factor model of Fama and French (2015), the investment CAPM of Hou et al. (2015), the mispricing model of Stambaugh and Yuan (2017), and the behavioral factor model of Daniel et al. (2020). The first five models were originally motivated by investors acting rationally, whereas the last two were motivated by investors acting irrationally. I will interpret all models from a rational perspective. In particular, I will say that a stock has a high model-implied risk if it has a high model-implied expected return.

Factor-based asset pricing models try to approximate the stochastic discount factor (SDF) as a function of a small number of pricing factors. If an SDF exists, the expected excess returns of a stock can be expressed as

$$E_t[r_{i,t+1}] - r_{f,t+1} = -(1 + r_{f,t+1})\text{cov}_t(m_{t+1}, r_{i,t+1}), \tag{1}$$

where $E_t[r_{i,t+1}]$ is the stock's expected return, $r_{f,t}$ is the risk-free rate, and m_t is the SDF. The risk-free rate is constant across stocks, so risk is simply the covariance between the stock's return and the SDF. Hansen and Jagannathan (1991) showed that the minimum-variance SDF is the portfolio with the highest Sharpe ratio—the tangency portfolio—so we can equivalently think of risk as the covariance between the stock's return and the return of the tangency portfolio.

Therefore, I start by computing the model-implied tangency portfolio. For each model, I create the model's pricing factors from scratch, following the original authors' implementation closely (Appendix A.3.1 describes the pricing factor construction). The tangency portfolio is then the combination of these factors with the highest realized Sharpe ratio from 1972 to 2021. For example, for the Fama and French (1993) 3-factor model, the tangency portfolio allocates 49% to the market factor, 41% to the value factor, and 10% to the size factor (Appendix Figure A.2 show the tangency portfolio weights for all models).

To find the conditional covariance between a stock and the tangency portfolio, I also need an estimate of the conditional variance-covariance matrix. I build this estimate each month using the methodology in Jensen et al. (2024), which in turn is inspired by the methodology used by MSCI Barra. Briefly, the method assumes that returns follow a factor structure based on 12 industry factors, 13 characteristic factors (such as value and momentum), and an idiosyncratic residual. One benefit of this approach is that it ensures a full rank variance-covariance matrix even though the number of stocks exceeds the number of time periods in the estimation period.[8]

The risk of a stock implied by a model k is then:

$$\text{Factor risk:} \qquad \text{mrisk}_{i,t}^{k} = e_{i,t}' \hat{\Sigma}_t \pi^k = \text{cov}_t(r_{i,t+1}, r_{t+1}^{k}), \tag{2}$$

where $\text{mrisk}_{i,t}^{k}$ is the model-implied risk of the stock, $e_{i,t}$ is a conformable vector with 1 in

[8] An alternative approach is to estimate the conditional covariance between the tangency portfolio and the stock by using a rolling historical covariance over, say, the past 5 years. However, this approach assumes that the historical covariance is a good proxy for the future covariance, which is difficult to justify for individual stocks. For example, Facebook's conditional covariance will probably change as it moves from a newly IPO'ed growth stock to a more established mega-cap. My approach ensures that Facebook's conditional covariance with the tangency portfolio depends on its observable characteristics at each point in time.

the ith position and zero elsewhere, $\hat{\Sigma}_t$ is the variance-covariance estimate, π^m is the model-implied tangency portfolio, and r_t^k is the return of this portfolio. I standardize the $\text{mrisk}_{i,t}^k$ estimates each month by subtracting the cross-sectional mean and dividing by the cross-sectional standard deviation (to make them comparable to the machine learning estimates from the next section).

1.2.2 Machine learning risk

Rather than approximating the SDF with pricing factors, another approach is to predict realized returns directly. If investors have rational expectations, then (1) shows that any variation in predicted returns must come from variation in risk. In other words, a good model of realized returns is a good model of risk. Gu et al. (2020) shows that various machine learning (ML) methods are particularly suitable for building good models of realized returns. I use the XGBoost algorithm from Chen and Guestrin (2016), which is based on an ensemble of gradient-boosted decision trees and has been shown to work well on tabular data (for example, Shwartz-Ziv and Armon (2022) shows that XGBoost outperforms deep learning on a range of tabular benchmark tasks).

The dependent variable is the realized excess return of a stock in the next month, and the independent variables are the 153 stock characteristics from Jensen et al. (2023). The first model, $\hat{f}_1(x_{i,t})$, is trained using data from 1952 to 1971, where I use the last ten years to choose hyper-parameters. This model is used to predict realized returns from 1972 to 1981. I then estimate a new model, $\hat{f}_2(x_{i,t})$, using training data from 1952 to 1981, and use this model to predict realized returns from 1982 to 1991. I continue this process each decade and use the resulting five models to generate predicted returns from 1972 to 2021. Appendix A.3.3 provides additional details on the model fitting process.

The risk of a stock as implied by the model is then:

$$\text{ML risk:} \qquad \text{mrisk}_{i,t}^k = \hat{f}_{t<h}(x_{i,t}), \tag{3}$$

where the notation $\hat{f}_{t<h}$ indicates that I use the most recent model h estimated on training data before t. In other words, all the predictions are made out-of-sample. Similar to the approach for the subjective and factor risk estimates, I standardize the ML risk estimates each month by subtracting the cross-sectional mean and dividing by the cross-sectional standard deviation. Table 2 shows the correlation between the different risk measures, computed each month and averaged over time.

Table 2: Correlation Matrix

	VL	MS	CAPM	FF3	CH4	FF5	HXZ4	SY4	DHS3	ML
VL	1.00									
MS	0.84	1.00								
CAPM	0.45	0.41	1.00							
FF3	0.43	0.43	0.97	1.00						
CH4	0.11	0.18	0.50	0.50	1.00					
FF5	0.23	0.21	0.75	0.77	0.52	1.00				
HXZ4	0.18	0.17	0.68	0.70	0.56	0.92	1.00			
SY4	0.07	0.07	0.55	0.54	0.57	0.77	0.76	1.00		
DHS3	0.03	0.06	0.62	0.60	0.56	0.68	0.67	0.66	1.00	
ML	-0.20	-0.13	-0.18	-0.18	0.15	-0.01	0.03	0.14	0.05	1.00

Note: The table shows the correlation between the two subjective risk measures and the eight model-based risk measures. The correlation is computed separately each month and then averaged over time.

2 Do asset pricing models capture risk or mispricing?

To guide the empirical analysis, I start by presenting a simple conceptual framework designed to show the problem with interpreting asset pricing models based on realized returns and then showing that this problem is alleviated with observable risk and mispricing proxies. Consider an economy with one representative investor, a risk-free rate of zero, multiple stocks indexed by i, and two periods, t when the stock prices $p_{i,t}$ are determined and $t+1$ when the stocks pay an uncertain dividend, $d_{i,t+1}$.

The price of a stock depends on the representative investor's subjective dividend expectation $\tilde{\mathrm{E}}_t[d_{i,t+1}]$ less a discount for their subjective risk of owning the stock, $\tilde{\mathrm{risk}}_{i,t}$, times a subjective price of risk that is constant across stocks, $\tilde{\lambda}_t$:

$$p_{i,t} = \frac{\tilde{\mathrm{E}}_t[d_{i,t+1}]}{1 + \tilde{\lambda}_t \tilde{\mathrm{risk}}_{i,t}}. \tag{4}$$

The representative investor's subjective expected return for a specific stock is:

$$\tilde{\mathrm{E}}_t[r_{i,t+1}] = \frac{\tilde{\mathrm{E}}_t[d_{i,t+1}]}{p_{i,t}} - 1 = \tilde{\lambda}_t \tilde{\mathrm{risk}}_{i,t}. \tag{5}$$

From the perspective of the representative investor, the subjective expected return of a stock only reflects its risk.

The objective expected return, however, can also reflect mispricing if the subjective dividend expectation differs from the objective one. Specifically, let $\mathrm{E}_t[d_{i,t+1}]$ denote the objectively correct dividend expectation, in which case the objective expected return is:[9]

$$\mathrm{E}_t[r_{i,t+1}] = \frac{\mathrm{E}_t[d_{i,t+1}]}{p_{i,t}} - 1 \tag{6}$$

$$\approx \underbrace{\tilde{\lambda}_t \tilde{\mathrm{risk}}_{i,t}}_{\text{risk } i} + \underbrace{\frac{\mathrm{E}_t[d_{i,t+1}] - \tilde{\mathrm{E}}_t[d_{i,t+1}]}{\tilde{\mathrm{E}}_t[d_{i,t+1}]}}_{\text{mispricing } i} \tag{7}$$

In addition to risk, the objective expected return also reflects whether the subjective cash flow is too high or low relative to the objective expectation.

When testing asset pricing models' ability to explain average realized returns, we essentially evaluate their ability to explain objective expected returns. Equation (7), however, shows that objective expected returns cannot be used to distinguish between whether a model captures risk or mispricing. For example, if value stocks have a 5% higher realized return

[9] The exact expression for the objective expected return is $\mathrm{E}_t[r_{i,t+1}] = \frac{\mathrm{E}_t[d_{i,t+1}]}{\tilde{\mathrm{E}}_t[d_{i,t+1}]}(1 + \tilde{\lambda}_t \tilde{\mathrm{risk}}_{i,t}) - 1$. The approximation is easier to understand, but the interpretation is the same: objective expected returns can reflect risk and/or mispricing.

than growth stocks, it could be because they are perceived as relatively risky or because they are undervalued relative to growth stocks. To distinguish between the two components, we need observable proxies for risk and mispricing. Continuing the example, if value stocks and growth stocks are perceived as equally risky we can reject the idea that the value premium is due to risk.

2.1 Asset pricing models and realized returns

I start by showing that newer models are better at explaining realized returns by estimating the following regression model:

$$r_{i,t+1} = \alpha_t + \beta \, \text{mrisk}_{i,t} + \epsilon_{i,t}, \tag{8}$$

where $r_{i,t+1}$ is the realized excess return, α_t is a year-month fixed effect, and β is the coefficient of interest. The residual errors $\epsilon_{i,t}$ are likely correlated across stocks at the same point in time, so I cluster standard errors by stock and year-month. I estimate the regression for non-microcap stocks, i.e., stocks with a market cap above the 20th percentile of NYSE stocks, because the coverage universe of Value Line and Morningstar is heavily tilted toward larger stocks. I also estimate the regression separately within mega, large, and small-cap stocks (Morningstar mainly cover mega-cap stocks, while Value Line mainly cover mega- and large-cap stocks), as defined in Jensen et al. (2023).

Figure 2 shows the estimated β along with 95% confidence intervals. The models are sorted from left to right by their β coefficient when estimated for all stocks. For all stocks, the CAPM and FF3 are the worst models at explaining realized returns, and in fact their estimated coefficients are negative. In other words, stocks with high CAPM or FF3 risk have lower realized returns on average than those with low risk. By contrast, newer empirical factor models have positive coefficients, with DHS3, CH4, and SY4 reaching statistical significance at a 5% level. The best-performing model is the ML model, where a one-

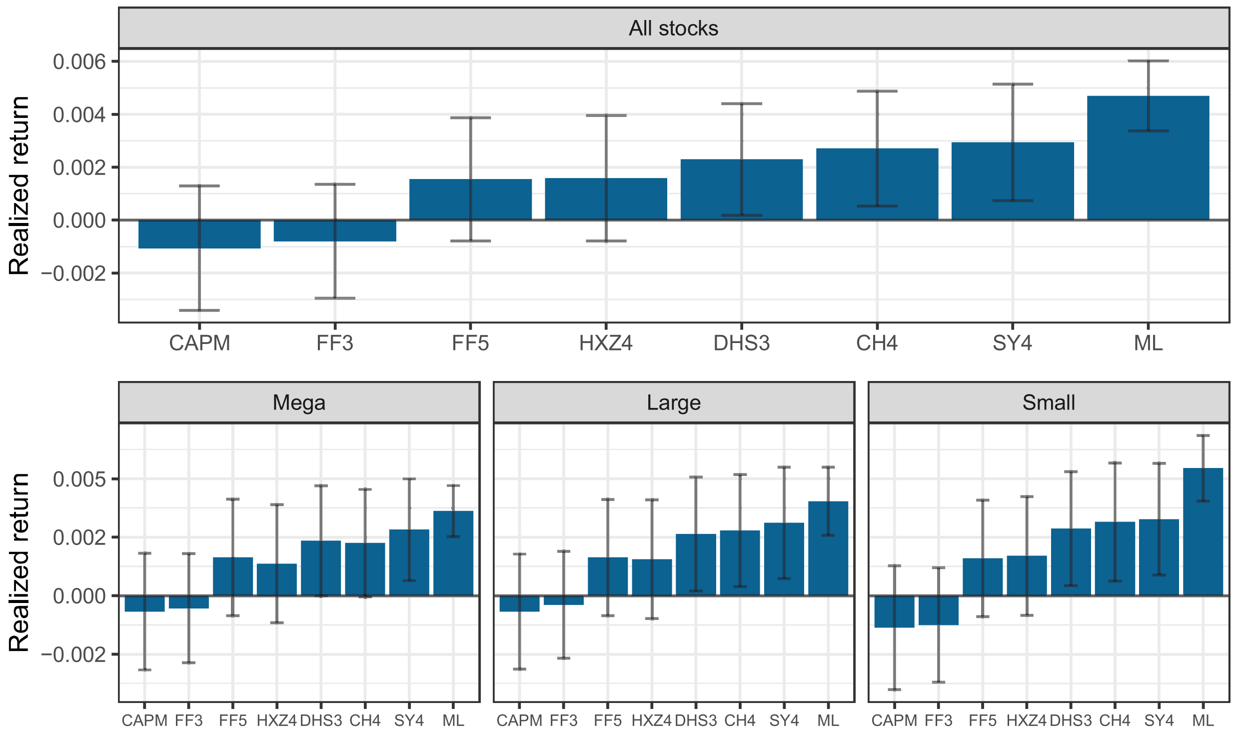


Figure 2: Realized returns

Note: The bars show the slope coefficient from regressing a stock's realized excess return in month $t+1$ on its model-implied risk in month t, that is, β from the regression in (8). The error bars show the corresponding 95% confidence interval. The top panel shows the coefficients estimated from the full sample of all non-micro-cap stocks, while the bottom panels show the coefficients estimated separately on mega-, large-, and small-cap stocks. The classification of stocks into size groups comes from Jensen et al. (2023).

standard-deviation increase in ML risk leads to an increase in the next month's realized return of 0.47%, which translates into 5.6% per year. The results are similar across mega, large, and small-cap stocks.

My approach to testing asset pricing models at the individual stock level is new. The standard approach is to test factor-based models on their ability to explain the realized returns of portfolios, for example, using the test from Gibbons et al. (1989, GRS), which Barillas and Shanken (2017) showed is equivalent to ranking asset pricing models by the Sharpe ratio of their tangency portfolio. Figure A.3 in the appendix shows that the ranking based on the tangency portfolio Sharpe ratio is very similar to the ranking from Figure 2.

2.2 Asset pricing models and risk

The ranking of models based on realized returns is informative about the models' ability to explain objective expected returns. However, as shown in the beginning of this section, the ability to explain objective expected returns cannot be used to infer whether the models capture risk or mispricing. The same point is raised by Kozak et al. (2018). In this subsection, I provide a more direct test of the models' ability to explain risk, as proxied for by subjective risk ratings and subjective expected returns from equity analysts.

2.2.1 Risk proxy I: Subjective risk

To investigate the models' ability to explain subjective risk, I estimate the following regression model:

$$\text{srisk}_{i,t} = \beta\, \text{mrisk}_{i,t} + \epsilon_{i,t}, \tag{9}$$

where $\text{srisk}_{i,t}$ is either the safety rank from Value Line or the cost of equity from Morningstar. The model does not have an intercept because both srisk and mrisk are standardized to have a mean of zero. The residual errors $\epsilon_{i,t}$ are likely correlated over time for the same stock and across stocks at the same point in time, so I cluster standard errors by stock and year-month. The model-implied risk measures are standardized, so models with a higher absolute β estimate will also have a higher R^2.

Figure 3 shows the estimated β along with 95% confidence intervals. The models are sorted from left to right by their ability to explain realized return. The left panel shows the ability of the models to explain the safety rank from Value Line. The CAPM and FF3 are by far the best. For example, a one standard deviation change in the CAPM-implied risk (the stock's market beta) leads to a 0.42 standard deviation change in the safety rank. The corresponding R^2 is 0.17. The third best model is FF5, but it is far less important with a β

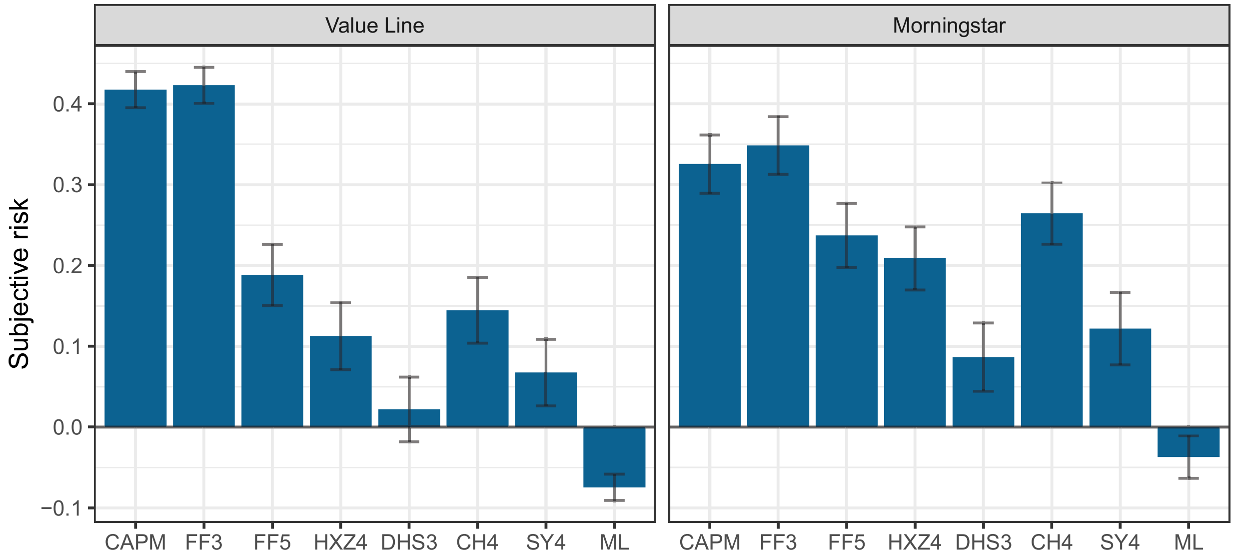


Figure 3: Subjective risk

Note: The bars show the slope coefficient from regressing a stock's subjective risk in month t on its model-implied risk in month t, that is, β from the regression in (9). The error bars show the corresponding 95% confidence interval. The left panel is based on the safety rank from Value Line, and the right panel is based on the cost of equity from Morningstar.

estimate of 0.19 and an R^2 of 0.03. The worst model for explaining subjective risk is ML, which has a β estimate of -0.07 and an R^2 of approximately zero. The relative ranking is similar in the right panel that uses the cost of equity from Morningstar. CAPM and FF3 are the best; ML is the worst. The outperformance of the CAPM and FF3 relative to the other models, however, is smaller.

Notably, the seven factor model all have a positive correlation with subjective risk. This finding mostly reflects the fact that all of the factor models I consider include a market factor, which is an important determinant of the conditional covariance for any stock. The market factor is typically unimportant for predicting future returns, but it is nevertheless often included in factor models because it is an important driver of contemporaneous returns (e.g., Hou et al. (2015, p. 663)). By contrast, ML is the only model with a negative correlation to subjective risk—probably because it is not forced to include a market factor.

Despite the fact that CAPM and FF3 explain subjective risk well, they do not explain it perfectly. This finding opens up the possibility that investors' risk perceptions are driven by effects not captured by current asset pricing models. In Appendix A.6, I regress the subjective risk ratings from Value Line and Morningstar on eight prominent security characteristics to understand the drivers of subjective risk. I find that subjectively risky stocks tend to be small (low market equity), volatile (high market beta and non-market volatility), and distressed (low Altman Z-score). Many existing factors models imply that high beta and small stocks are risky, but none of the models I consider include a volatility or a distress factor.[10]

Overall, Figure 3 shows one of the main results of the paper. Newer empirical asset pricing models like FF5 and ML explain realized returns well, even though they explain subjective risk poorly. Conversely, the CAPM and FF3 explain realized returns poorly even though they they explain subjective risk well. This result is evidence against the rational interpretation that newer models better capture risk.

2.2.2 Risk proxy II: Subjective expected return

My second proxy for risk is subjective return expectations from I/B/E/S, Value Line, and Morningstar. The idea is that, from the perspective of the representative investor, variation in subjective expected returns only reflect risk, as shown in (5). In practice, I do not have return expectations from the representative investor, so I use return expectations from equity analysts as a proxy.[11]

To investigate the asset pricing models' ability to explain subjective expected return, I

[10] Empirically, stocks with high volatility (Ang et al., 2006) or high distress risk (Campbell et al., 2008) tend to have low realized returns, which means that they cannot be a risk factor if investors have rational beliefs. My results, however, suggest that they do indeed represent risk from the perspective of analysts, suggesting that their low realized returns are a result of mispricing.

[11] One issue, though, is that the return expectations for any individual investor (or analysts) also reflect their view on whether the stock is mispriced. For example, the correlation between Morningstar's cost of equity (their subjective risk assessment) and their subjective expected return is only 0.09, so perceived mispricing drives most of the variation in subjective expected returns. As such, subjective expected returns are a noisy proxy for subjective risk. The benefit of using subjective expected return is that it is also available from I/B/E/S.

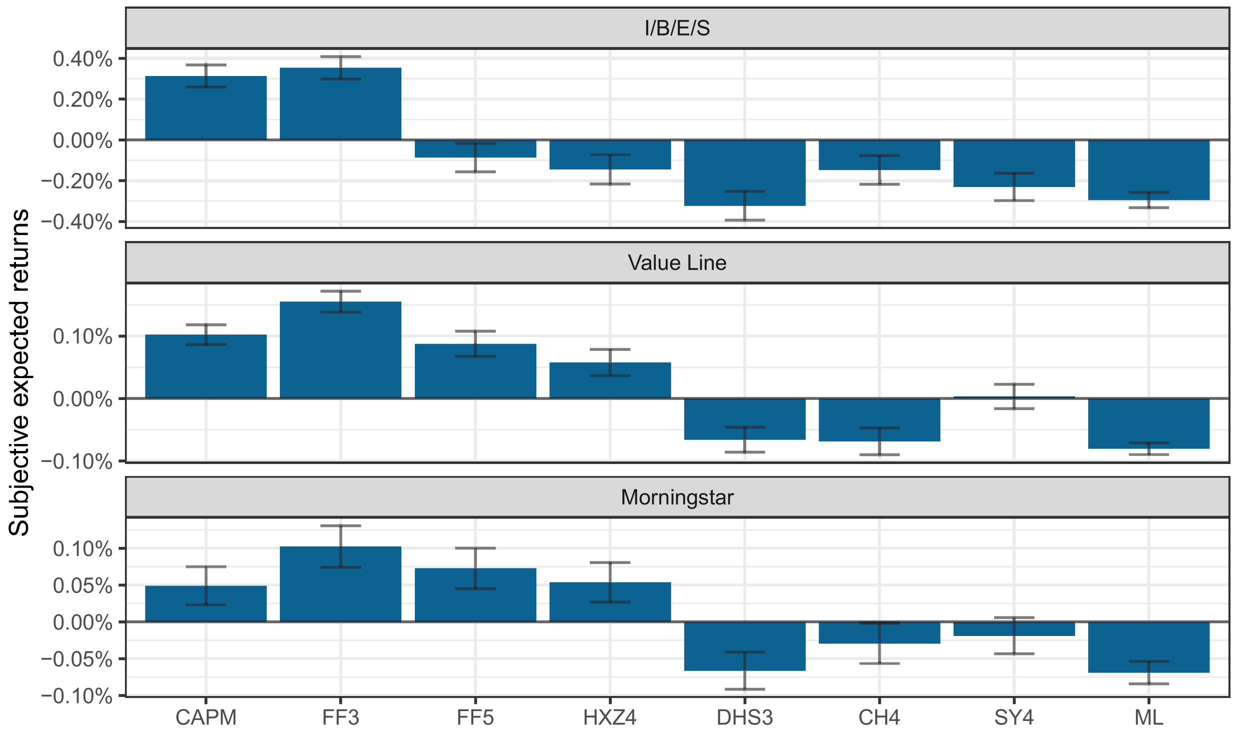


Figure 4: Subjective expected return

Note: The bars show the slope coefficient from regressing a stock's subjective expected return in month t on its model-implied risk in month t, that is, β from the regression in (10). The error bars show the corresponding 95% confidence interval. The top, middle, and bottom panels are based, respectively, on the one-year forecast from I/B/E/S, the four-year forecast from Value Line, and the three-year forecast from Morningstar.

estimate the following regression model:

$$\tilde{\mathrm{E}}_t[r_{i,t+1}] - r_{f,t+1} = \alpha_t + \beta\,\mathrm{mrisk}_{i,t} + \epsilon_{i,t}, \tag{10}$$

where $\tilde{\mathrm{E}}_t[r_{i,t+1}]$ is the subjective expected return over the next month from either I/B/E/S, Value Line, or Morningstar. I cluster the standard errors by stock and year-month.

Figure 4 shows the estimated β along with 95% confidence intervals. Overall, the CAPM, and especially FF3, are the best models for explaining subjective expected returns, which is a further testament to their ability to explain risk. This result is most pronounced when using return expectations from I/B/E/S, where the CAPM and FF3 are the only models with a positive parameter estimate. For all other models, a higher model-implied risk is associated with a lower subjective expected return from I/B/E/S. The worst-performing model is again the ML model. This finding is particularly striking, considering that ML is the best model of realized returns and, by extension, of objective expected returns. It highlights why realized returns should not be used to learn about risk.

2.3 Asset pricing models and mispricing

2.3.1 Mispricing proxy I: Earnings forecast errors

If models that explain realized returns well explain risk poorly, this disconnect must be explained by mispricing. I now test this idea directly. My first mispricing proxy is based on equation (7), which shows that a stock is mispriced if investors' subjective cash flow expectations differ from the objectively correct expectation. To proxy for investors' subjective expectations, I use cash flow forecasts from I/B/E/S and Value Line.

From I/B/E/S, I collect median earnings per share (EPS) forecasts from the consensus file over multiple horizons, ranging from the next fiscal quarter to the next three fiscal years. From Value Line, I collect their EPS forecast for the next five fiscal years. Following Bouchaud et al. (2019), I only retain the first forecast about a particular firm-fiscal end (e.g., Apple in Q1-2020) made at least 45 days and at most 180 days after the last fiscal year-end. Appendix A.2.3 describes the data collections in more detail.

For each forecast, I compute the realized forecast error as:

$$\text{FE}_{i,t,h} = \frac{\text{EPS}_{i,t+h} - \tilde{\text{E}}_t[\text{EPS}_{i,t+h}]}{p_{i,t}}, \tag{11}$$

where $\text{EPS}_{i,t+h}$ is the realized EPS in fiscal end $t+h$, $\tilde{\text{E}}_t$ is the consensus forecast from I/B/E/S or Value Line, and $p_{i,t}$ is the stock price at the time of the forecast (the price and

EPS are adjusted for stock splits). Scaling by the stock price ensures that the forecast error is relatively stationary across stocks. A positive forecast error means that the realized cash flow was higher than analysts expected, and vice versa for a negative forecast error.

To test the asset pricing models' ability to explain biased cash flow expectations, I estimate the following regression model:

$$\text{FE}_{i,t,h} = \alpha_t + \beta\, \text{mrisk}_{i,t} + \epsilon_{i,t+h}, \tag{12}$$

where α_t is a fiscal-year-quarter end fixed effect (i.e., a constant for all firms whose fiscal period ends in the same year and quarter). Standard errors are clustered by firm and fiscal-year-quarter end.

Figure 5 shows the estimated β along its 95% confidence interval across the different asset pricing models, forecast providers, and forecast horizons. The blue bars show the estimates using expectations from I/B/E/S. The results are remarkably consistent across all specifications. The CAPM and FF3 always have a negative estimate, suggesting that stocks they imply to be risky have excessively high subjective cash flow expectations. The negative estimate is significant at all horizons above two quarters ahead, and the magnitude of the estimate increases with the forecast horizon. For the CAPM, a one standard deviation increase in market beta is associated with a one-quarter ahead forecast that is 0.01% too high (relative to the stock price), a one-year ahead forecast that is 0.23% too high, and a three-year ahead forecast that is 0.74% too high. The corresponding numbers for FF3 are similar but even larger in magnitude.

These high cash flow expectations can explain how the CAPM and FF3 can explain ex-ante subjective risk but not ex-post realized returns. For example, consider a low and a high beta stock with a required return of, respectively, 5% and 10%. Suppose the low beta stock has no cash flow surprise but the high beta stock has a negative surprise that leads to a negative return of −5%. Ex-ante investors required a higher return for investing in the high

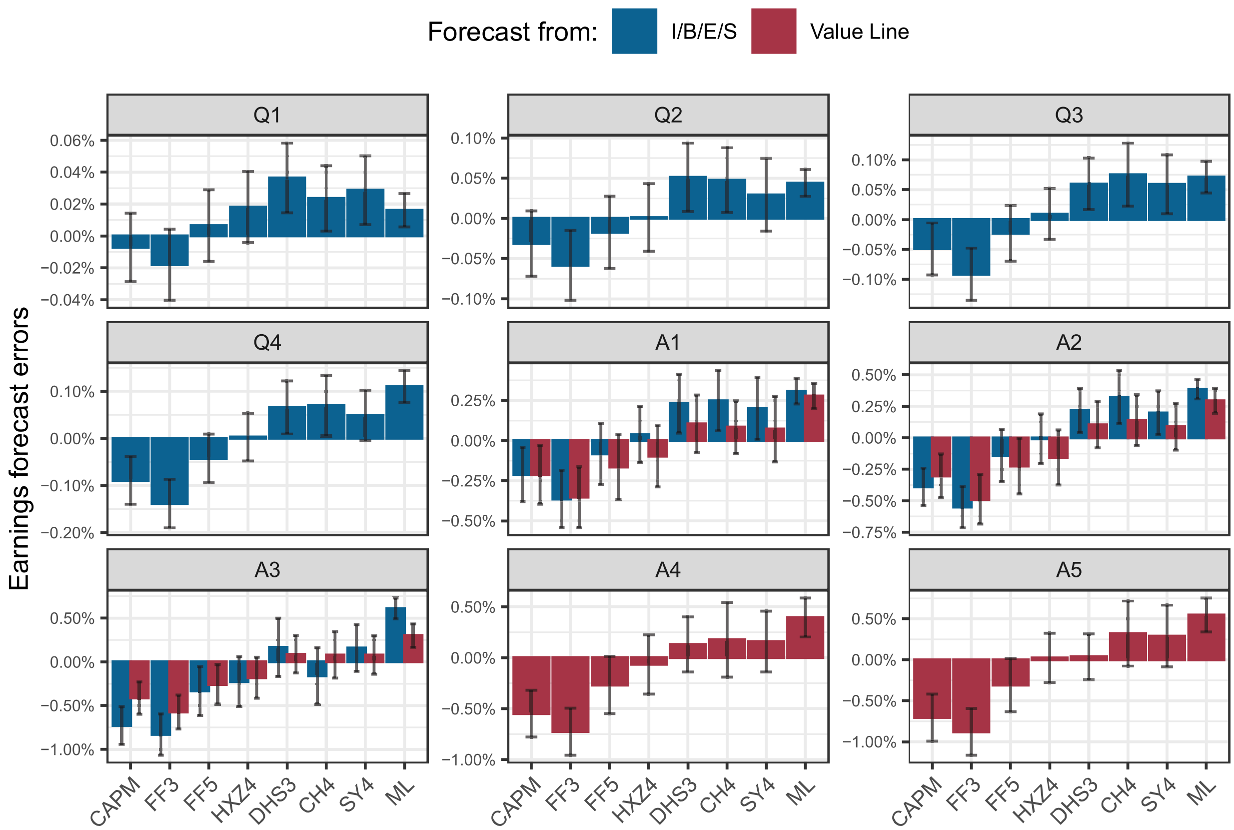


Figure 5: Earnings forecast errors

Note: The bars show the slope coefficient from regressing a stock's earnings forecast error (the realized earnings at $t + h$ minus the expected earnings at t scaled by the stock price at t) on its model-implied risk in month t, that is, β from the regression in (12). The error bars show the corresponding 95% confidence interval. The different panels show forecast errors over different forecast horizons ranging from one-quarter ahead (Q1) to five years ahead (A5). The color of the bar indicates whether the expected earnings come from I/B/E/S or Value Line.

beta stock due to its perceived risk, but ex-post, its realized return is exactly the same as that of the low beta stock.

Turning to the models that explain realized returns well, Figure 5 shows that the estimates for FF5 and HXZ4 are mostly insignificant. For the four remaining models, however, the parameter estimates are mostly positive and significant. None of the models are clearly more strongly related to forecast errors than the others. For example, Sy4 has the strongest relationship with one-quarter ahead forecasts, the four models have a similar relationship with one-year ahead forecasts, and ML has the strongest relationship with three-year ahead forecasts. As for CAPM and FF3, the magnitude of the estimates tends to increase with the forecast horizon, although this pattern is less pronounced. Overall, these findings suggest that newer models explain realized returns well, at least in part, because they are able to identify stocks that are mispriced.

I/B/E/S expectations primarily come from sell-side analysts employed by banks, and one commonly raised concern is that their expectations reflect incentives rather than true beliefs (see, e.g., Michaely and Womack (1999)). For example, analysts' expectations could reflect their desire to generate trading commissions or to cater to potential M&A clients. To alleviate this concern, I also use cash flow expectations from Value Line. Value Line does not receive any money from trading commissions or M&A activity; rather, their main source of revenue is their investment research. Value Line analysts, therefore, have much better incentives to align their forecast with their true beliefs. Luckily, the results using Value Line expectations are highly similar to those using I/B/E/S expectations. Value Line analysts are too optimistic about stocks that the CAPM and FF3 deem risky and too pessimistic about stocks that models that newer models deem risky.

2.3.2 Mispricing proxy II: Earnings announcement returns

The results from the previous section rely on the assumption that the cash flow expectations from I/B/E/S and Value Line are a good proxy for expectations from the representation investor. It also rests on the assumption that beliefs translate strongly into investors' portfolio allocations, which has been questioned by Giglio et al. (2021) and Charles et al. (2022). To ensure that the biased beliefs translate into biased prices, I, therefore, also use the return of a stock on its earnings announcement day as my second proxy for mispricing. Earnings announcement days contain substantial firm-specific news, so the idea is to assume that the release of firm-specific news partially corrects mispricing from biased beliefs. For example, a negative earning announcement return suggest that the announced earnings was lower than expected by the representative investor.

I get quarterly earnings announcement days and times from I/B/E/S, starting in 1989. Earnings announcements are typically made outside of market hours, so I define the earnings announcement day t as the first trading day after the announcement. I then regress the return on the earnings announcement (trading) day above a benchmark return on the subjective risk before the announcement:

$$r_{i,t} - r_{b,t} = \alpha + \beta \, \text{mrisk}_{i,t-21} + \epsilon_{i,t}, \tag{13}$$

where $r_{i,t}$ is the return on the earning announcement day, $r_{b,t}$ is the benchmark return, α is an intercept, and standard errors are clustered by the earnings announcement date. The benchmark return is either the risk-free rate, the market return, or the market return multiplied by the stock's beta computed on the 21 days before and after the earnings announcement day.

Figure 6 shows the β estimate and its 95% confidence interval. The results are similar across the different benchmark returns, so I will interpret the estimates for the bottom panel, where the benchmark is the market return times the stock-specific beta. The CAPM and FF3 are, again, the models with the most negative relationship to mispricing. A one standard deviation increase in CAPM risk, for example, is associated with a 0.16% lower excess return on the earnings announcement date. Assuming that the average year has 252 trading days, this number corresponds to an annualized return of 41.2%. Similarly, a one standard deviation increase in FF3 risk is associated with a 0.14% lower excess return (36.1% annualized).

For the remaining factor models, the estimate is generally positive but not statistically significant regardless of the benchmark return. By contrast, the estimate for ML is positive

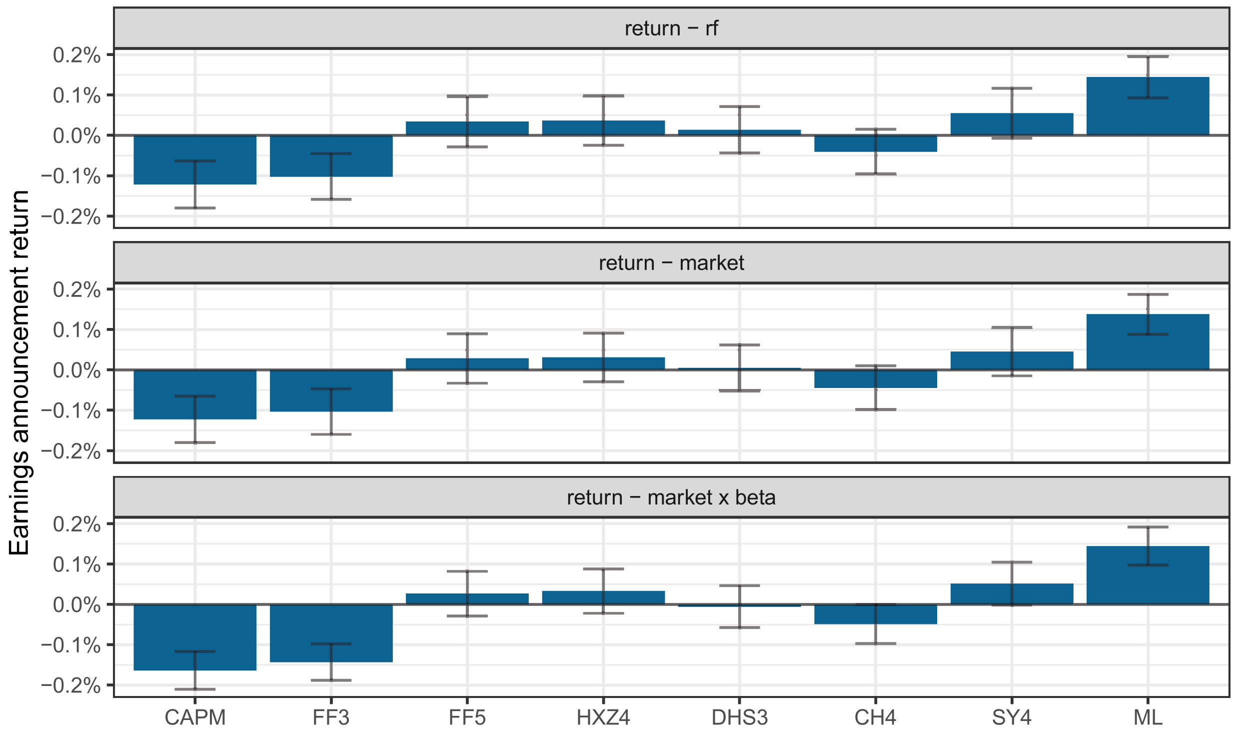


Figure 6: Earnings announcement returns

Note: The bars show the slope coefficient from regressing a stock's benchmark-adjusted return during its earnings announcement day on its model-implied risk in the month before the announcement, that is, β from the regression in (13). The error bars show the corresponding 95% confidence interval. The benchmark in the top, middle, and bottom panels are, respectively, the risk-free rate, the market return, and the market return times the stock's market beta computed on the 21 days before and after its earnings announcement.

and hugely statistically significant. A one standard deviation increase in ML risk is associated with a 0.14% higher return on the earnings announcement day (36.3% annualized). Overall, the evidence is consistent with traditional models suffering a drag on their realized return due to mispricing while newer models are getting a boost.

The results imply that mispricing resolved during earnings announcements has a sizable effect on realized returns. But how big is the effect of mispricing on realized returns over a full year? At one extreme, if there is no cash flow news outside of earnings announcements, the estimates should be multiplied by four (because most stocks have four earnings announcements per year), and a one standard deviation increase in CAPM risk leads to

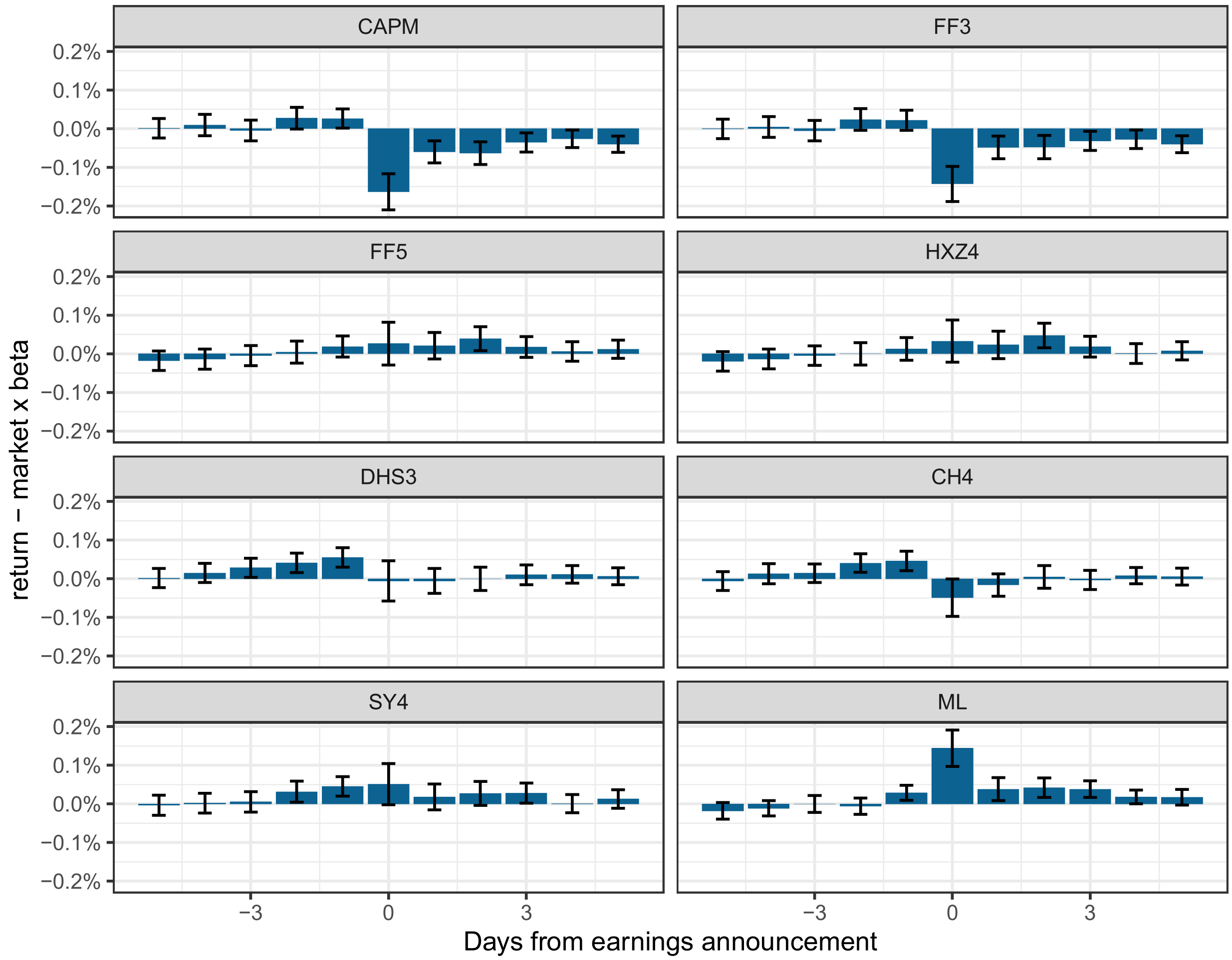


Figure 7: Returns around earnings announcements

Note: The bars show the slope coefficient from regressing a stock's benchmark-adjusted return on the 5 days before, the day of, and the five days after its earnings announcement day on its model-implied risk in the month before the announcement. The error bars show the corresponding 95% confidence interval. The benchmark is the market return times the stock's market beta computed on the 21 days before and after its earnings announcement.

a return-subtracting mispricing of 0.65%. At another extreme, if cash flow news arrives with equal intensity outside of earnings announcement, the number should be multiplied by 252, which would imply that a standard deviation increase in CAPM risk leads to return-subtracting mispricing of 41.2%.

To infer the intensity of news outside of the exact earnings announcement day, Figure

7 shows the realized alpha (stock return minus market return times beta) on the 5 days before, the day of, and the 5 days after the earnings announcement. In other words, I estimate the regression from (13) eleven times with the realized alpha on each day as the dependent variable. If there's no cash flow news outside of the earnings announcement days, all estimates outside of the earnings announcement day should be insignificant. If cash flow news arrives with equal intensity outside of the earnings announcement day, all estimates should be the same. The actual estimates are somewhere in between. The estimate on the earnings announcement days are most often two to four times larger than the second-largest estimate, suggesting that the earnings announcement day is special. There are, however, also plenty of significant estimates outside of the earnings announcement day, suggesting that these days also contain cash flow news.

For the CAPM, the estimates on the five days before the earnings announcement date are insignificant and small, but the estimates on the five days after are significant and large.[12] Summing the estimates over the eleven days around the earnings announcement gives 0.33%, which, with four earnings announcements per year, gives an annual return drag of 1.3% for each standard deviation increase in CAPM risk. The corresponding difference in realized return between a low beta stock 2 standard deviations below the average and a high beta stock 2 standard deviations above average is 5.3%. For ML, the equivalent return boost per standard deviation is 1.2%, which amounts to a high-low difference of 4.7%. As such, mispricing resolved during the few days around earnings announcements can create a substantial wedge between the ex-ante required and ex-post realized return of a stock, which can explain why, say, the CAPM explains realized returns poorly even though it explains required returns well.

[12]Note that the CAPM estimates on the two days before the earnings announcement date is almost significantly *positive* rather than negative. I do not analyze this finding further, but it is consistent with investors having lottery preferences (Barberis and Huang, 2008), whereby they allocate to high beta stocks immediately before the earnings announcement day because they expect high beta stocks to have volatile earnings announcement returns.

3 Do equity factors capture risk or mispricing?

The previous sections used observable risk and mispricing proxies to interpret asset pricing models. This section uses the same proxies to interpret the 119 equity factors studied by Jensen et al. (2023).[13] Rational factor theories imply that equity factors have positive realized returns because the underlying factor characteristic, for example, the book-to-market characteristic that underlies the value factor, capture risk (see, e.g., Berk et al. (1999), Carlson et al. (2004), Zhang (2005), Lettau and Wachter (2007), and Kogan and Papanikolaou (2013, 2014)). Behavioral factor theories imply that equity factors have positive realized returns because the underlying factor characteristic captures mispricing (see, e.g., Shefrin and Statman (1985), De Long et al. (1990a,b), Daniel et al. (1998), Barberis et al. (1998), Hong and Stein (1999), Barberis and Huang (2008), Hong and Sraer (2016), and Bordalo et al. (2019)).

To distinguish between these two interpretations, I estimate four regressions for each of the 119 factors:

$$\begin{aligned}
\text{srisk}^{\text{vl}}_{i,t} &= \alpha^{\text{vl}}_t + \beta^{\text{vl}} x_{i,t} + \epsilon^{\text{vl}}_{i,t}, &(14)\\
\text{srisk}^{\text{ms}}_{i,t} &= \alpha^{\text{ms}}_t + \beta^{\text{ms}} x_{i,t} + \epsilon^{\text{ms}}_{i,t}, &(15)\\
\text{FE}_{i,t,12} &= \alpha^{\text{fe}}_t + \beta^{\text{fe}} x_{i,t} + \epsilon^{\text{fe}}_{i,t}, &(16)\\
r_{i,t} - r_{b,t} &= \alpha_t + \beta^{\text{ear}} x_{i,t-1} + \epsilon^{\text{ear}}_{i,t}. &(17)
\end{aligned}$$

The first two regressions show the factor's relationship with risk by regressing subjective risk from Value Line and Morningstar, respectively, on the factor characteristic. The next two regressions show the factor's relationship with mispricing by regressing the next-year earnings forecast error and the earnings announcement return minus the market return times

[13] Jensen et al. (2023) studies 153 factors in total, but only 119 have a paper claiming that the factor is a significant predictor of returns.

the stock's beta, respectively, on the factor characteristic. The rational interpretation imply that β^{vl} and β^{ms} are positive, while the behavioral interpretation imply that β^{fe} and β^{ear} are positive.

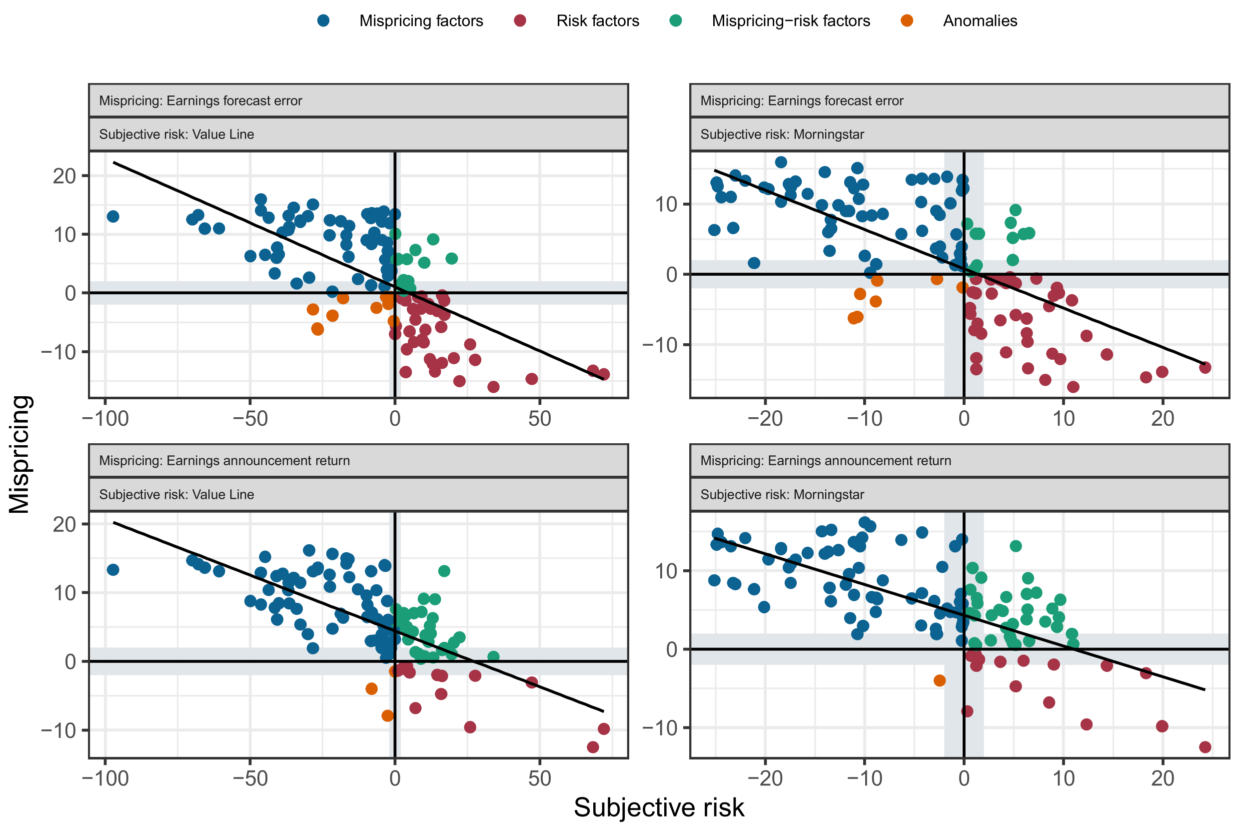


Figure 8: Risk and mispricing coordinates for equity factors

Note: The figure shows the risk and mispricing coordinates of 119 equity factors. The x-coordinate in the left and right column is, respectively, the t-statistic of the β^{vl} estimate from (14) and the t-statistic for the β^{ms} estimate from (15). The x-coordinate, therefore, shows the relationship between the factor characteristic and subjective risk. The y-coordinate in the top and bottom row is, respectively, the t-statistic of the β^{fe} estimate from (16) and the β^{ear} estimate from (17). The y-coordinate, therefore, shows the relationship between the factor characteristic and mispricing. The factors are colored according to the signs of their relationship with risk and mispricing. Points outside of the grey-shaded areas are statistically significant at the 5% level. The solid black line shows the best linear fit.

Figure 8 shows the results of these regressions by showing either β^{vl} or β^{ms} on the x-axis and β^{fe} or β^{ear} on the y-axis along with their 95% confidence intervals (Table A.4 in the appendix provides the exact risk and mispricing coordinates). The first takeaway is that the rational interpretation fails for most equity factors. Only 42% of β^{vl} and 43% of β^{ms} are positive and only 37% and 29%, respectively, are significantly positive at the 5% level. By contrast, the behavioral interpretation is more successful as 61% of β^{fe} and 86% of β^{ear} are positive, and 55% and 73% are significantly positive at the 5% level. These results suggest that more factors arise from behavioral mispricing than from rational compensation for risk.

To better interpret equity factors, I divide them into four groups, defined by the four quadrants in Figure 8. I use the following terminology: The top-left group are clear "mispricing factors" because they have a negative relationship with risk (so they cannot be explained by the rational interpretation) and a positive relationship with mispricing (so they can be explained by the behavioral interpretation); The bottom-right group are clear "risk factors" because they can only be explained by the rational interpretation; The top-right group are "mispricing-risk factors" because they are consistent with both the rational and behavioral interpretation; finally, the bottom-left group are "anomalies," because they cannot be explained by the rational or the behavioral interpretation.

Figure 9 shows the classification for the four combinations of risk and mispricing proxies. Mispricing factors are by far the largest group, with a share averaging 53% across all combinations. Prominent factors that are always in this group across the four combinations are the return momentum factor from Jegadeesh and Titman (1993), the low volatility factor from Ang et al. (2006), the low beta factor from Frazzini and Pedersen (2014), the low distress factor from Dichev (1998), the low equity issuance factor from Pontiff and Woodgate (2008), and the quality-minus-junk factor from Asness et al. (2019). Risk factors are the second largest group on average, with a share averaging 22%. Prominent factors that are always in this group are the size factor from Banz (1981) and the long-run reversal factor from De Bondt and Thaler (1985). Many factors in this group are size or liquidity-based, suggesting that investors require compensation for bearing size and liquidity risk but that their realized returns are too low because small illiquid stocks tend to have overoptimistic

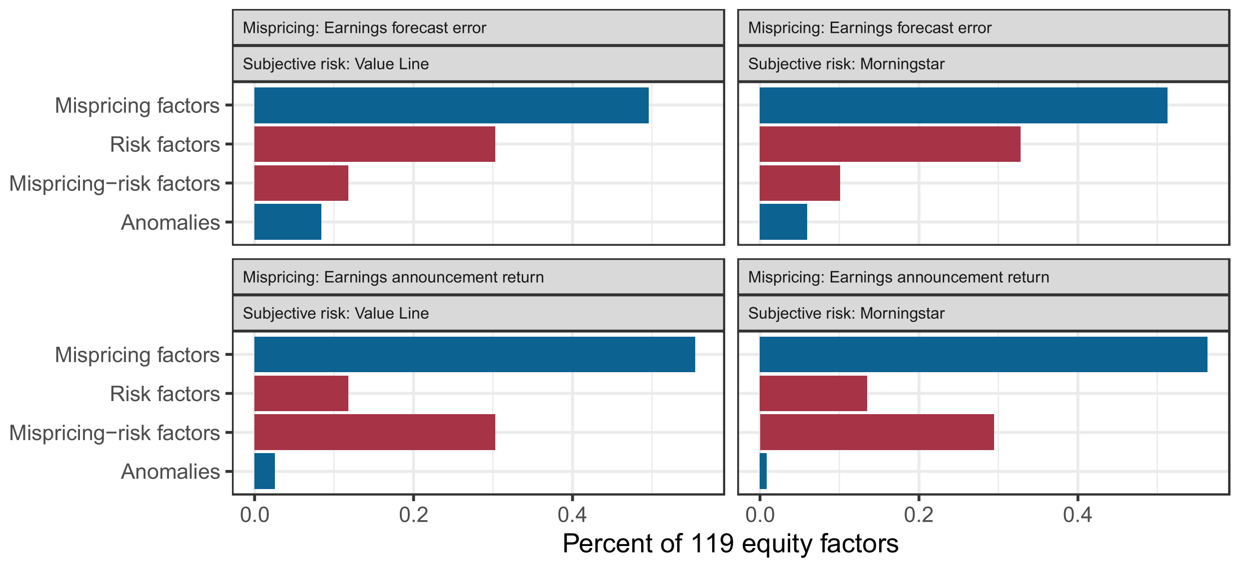


Figure 9: Equity factor classification

Note: The figure classifies equity factors into four groups, defined by the sign of the correlation between the factor characteristics and risk and mispricing proxies, and shows the share of each group. Mispricing factors have a negative correlation with risk and a positive correlation with mispricing; risk factors have a positive correlation with risk and a negative correlation with mispricing; mispricing-risk factors have a positive correlation with risk and mispricing; and anomalies have a negative correlation with risk and mispricing.

cash flow expectations. Mispricing risk factors are the third largest group with a share averaging 20%. Only seven out of the 119 factors are always in this group, among which the majority are related to debt issuance and inventory growth. Finally, the anomaly group is the smallest, with a share averaging 4%, and not a single factor is always in this group. The low share of the anomaly group suggests that the classification has succeeded in providing an interpretation of the vast majority of equity factors.[14]

Overall, the results suggest that in the zoo of equity factors, most reflect mispricing but some reflect risk. The results echo those for asset pricing models. Many equity factors were proposed because they could not be explained by the CAPM or FF3, but since these

[14] Some prominent factors that are not always in the same group are: the value factor from Rosenberg et al. (1985) (risk or mispricing-risk factor), the low asset growth factor from Cooper et al. (2008) (risk or mispricing-risk factor), and the gross profitability factor from Novy-Marx (2013) (mispricing or mispricing-risk factor).

traditional models do a good job of capturing risk, most of the apparent "anomalies" must capture mispricing.

4 The Optimism-Adjusted CAPM

The empirical results suggest that traditional models like the CAPM do a good job of capturing risk but fail to explain average realized returns because they also capture return-subtracting mispricing in the form of biased cash flow expectations. In this section, I argue that leading theoretical models designed to explain the CAPM failure cannot simultaneously explain these findings and propose a new model that can.

Models that explain the weak relationship between beta and realized returns with frictions cannot explain the strong relationship between beta and biased cash flow expectations. For example, Black (1972) and Frazzini and Pedersen (2014) consider investors with rational expectations, some of which are leverage-constrained. The leverage-constrained investors prefer investing in high beta stocks because they cannot lever up safe stocks, which leads to the weak relationship between beta and average realized returns. However, investors have rational expectations, so their cash flow expectations are unbiased.

Similarly, Miller (1977) and Hong and Sraer (2016) consider rational investors supplemented by an equal mass of optimists and pessimists who disagree about the expected market return. The optimists prefer investing in high beta stocks rather than levering up safe stocks because idiosyncratic variance is assumed to be constant across stocks. Due to short-selling constraints, pessimists fail to counteract the excess demand for high beta stocks which leads to the weak relationship between beta and average realized returns. Shorting constraints, however, do not affect expectations, and because there is an equal mass of optimists and pessimists, the cash flow expectation of the average investor is unbiased.

I propose a model that explains the weak relationship between beta and average realized return from biased beliefs rather than frictions. The model modifies the CAPM by assuming that investors with an optimism bias outnumber those with a pessimism bias, or, in other words, that optimists outnumber pessimists.

The idea that optimists outnumber pessimists is strongly supported by research in neuroscience and psychology. For example, Sharot (2011) reports that, across a range of forecasting methods and domains, most estimates imply that 80% of the population are too optimistic while the remaining 20% are too pessimistic. Optimism bias also manifests in other biases, such as overconfidence (the tendency to be overly confident about one's own forecasts) and the planning fallacy (the tendency to overestimate benefits and underestimate costs when making plans). Kahneman (2011, p. 255) argues that, in terms of its consequences for decision making, optimism bias could be the most important cognitive bias.[15] In finance, Brunnermeier and Parker (2005) show that an optimism bias is optimal for maximizing agents' subjective expected utility, and Cassella et al. (2023) document optimism bias in macroeconomic expectations. In short, prior research suggests that optimists outnumber pessimists.

4.1 Model setup

I consider an economy with two periods populated by investors $j = 1, \ldots, J$ with mean-variance preferences and absolute risk aversion γ^j. The investors trade securities $i = 1, \ldots, n$ at time 0. Each security has π_i^* shares outstanding and pays a liquidating dividend d_i at time 1. The dividend has an expected value of θ_i and variance of ω_i^2. The variance-covariance matrix of all dividends is denoted by Ω. For simplicity, I assume that investors know the dividend variance-covariance matrix but differ in their beliefs about the dividend expectation.

[15] Kahneman (2011, p. 255) writes: "The planning fallacy is only one manifestation of a pervasive optimistic bias. Most of us view the world as more benign than it really is, our own attributes as more favorable than they truly are, and the goals we adopt as more achievable than they are likely to be. We also tend to exaggerate our ability to forecast the future, which fosters optimistic overconfidence. In terms of its consequences for decision making, the optimistic bias may well be the most significant of the cognitive biases."

I denote the objectively correct expectation as E and the subjective expectation of investor j as $\tilde{\mathrm{E}}^j$. Finally, I assume that the risk-free rate is zero.

The prior distribution of θ_i, shared by all investors, is normal with a mean of μ_0 that is constant across stocks and a stock-specific variance of $\tau_0^2\omega_i^2$, where $\tau_0 > 0$. The prior variance is proportional to the dividend variance, which captures the intuition that uncertainty about the mean is higher for stocks with more variable dividends.

Investors observe two public signals for each stock, which are informative about the objective dividend expectation. The two signals are independently generated from a normal distribution with a mean of θ_i—the objective dividend expectation—and variance $\tau_1^2\omega_i^2$, where $\tau_1 > 0$. Again, the signal variance is proportional to the dividend variance, meaning that stocks with more variable dividends have more variable signals. Without loss of generality, I define s_i^{max} as the maximum of the two signals and s_i^{min} as the minimum.

4.2 Subjective dividend expectations with an optimism bias

To forecast the dividend of a stock, the rational (Bayesian) approach is to put equal weight on the two signals and some weight on the prior mean (all proofs are in Appendix A.1):

$$\mathrm{E}[\theta_i | s_i^{max}, s_i^{min}] = (1-\delta)\mu_0 + \frac{\delta}{2} s_i^{max} + \frac{\delta}{2} s_i^{min}, \tag{18}$$

where $\delta = \frac{\omega_i^2\tau_0^2}{\omega_i^2\tau_0^2 + \omega_i^2\tau_1^2/2} = \frac{\tau_0^2}{\tau_0^2 + \tau_1^2/2}$ is the shrinkage constant that determines the weight on the signals relative to the prior. The shrinkage constant is the same for all stocks because the prior and signal variances are both proportional to the stock's dividend variance.

To incorporate the possibility of irrational beliefs, I assume that the investor's actual beliefs are distorted relative to the rational benchmark:

$$\tilde{\mathrm{E}}^j[\theta_i | s_i^{max}, s_i^{min}] = (1-\delta)\mu_0 + \left(\frac{\delta}{2} + \kappa^j\right) s_i^{max} + \left(\frac{\delta}{2} - \kappa^j\right) s_i^{min}, \tag{19}$$

where κ^j is the investor-specific distortion. An optimistic investor has $\kappa^j > 0$, which means that the investor puts too much weight on the highest signal and too little weight on the lowest signal. This specification of optimism bias is consistent with the good news-bad news effect studied in Eil and Rao (2011) and the valence effect studied in Sharot and Garrett (2016), where people react more strongly to good news than bad news.

I define the bias in an investor's subjective dividend expectation as:

$$b_i^j = \tilde{\mathrm{E}}^j[\theta_i | s_i^{max}, s_i^{min}] - \mathrm{E}[\theta_i | s_i^{max}, s_i^{min}]. \tag{20}$$

This bias depends on two random signals, but Proposition 1 shows its expected value:

Proposition 1 (**Cash flow uncertainty leaves room for optimism**). *The subjective dividend expectation is:*

$$\tilde{\mathrm{E}}^j[\theta_i | s_i^{max}, s_i^{min}] = \mathrm{E}[\theta_i | s_i^{max}, s_i^{min}] + \mathrm{E}[b_i^j] + u_i^j, \tag{21}$$

where $u_i^j = b_i^j - \mathrm{E}[b_i^j]$ is mean zero noise coming from the randomness of the signals. The expected bias is:

$$\mathrm{E}[b_i^j] = c\kappa^j \omega_i, \tag{22}$$

where $c > 0$ is a positive constant. For an optimistic investor, $\kappa^j > 0$, the expected bias is positive and increase in the stock's dividend volatility.

Proposition 1 shows that the expected bias reflects the investor's optimism and the stock's dividend volatility. By putting too much weight on the highest signal, optimistic investors become overly optimistic about all stocks, but especially those with uncertain dividends.

To understand why optimistic investors are more optimistic about uncertain stocks, consider first an optimistic investor trying to forecast the cash flows of a firm with low uncertainty, such as a utility firm. Utility firms tend to have regulated pricing structures and long-term contracts, so the cash flow in "good" (s_i^{max}) and "bad" (s_i^{min}) states are similar. The forecast of the optimistic investors is, therefore, similar to that of the rational investor because the low cash flow uncertainty leaves limited room for optimism. In the extreme case where the cash flow is known (say, for a U.S. government bond), the forecast of the optimistic investor is equal to that of the rational investor.

By contrast, consider an optimistic investor trying to forecast the cash flows of a firm with high cash flow uncertainty, such as Tesla. In some states of the world, electric vehicles replace traditional cars, with Tesla as the market leader. In other states, Tesla goes bankrupt. As a result, Tesla leaves plenty of room for optimism, meaning that the expected bias for an optimistic investor is high.

Appendix A.4 shows that high beta stocks tend to have much more uncertain cash flows than low beta stocks. Proposition 1 can, therefore, explain why high beta stocks have relatively more optimistic cash flow expectations than low beta stocks.

4.3 Equilibrium with optimism bias

To show the effect of optimism bias on prices and returns in equilibrium, I assume that investors' dividend expectations follow (19), except that I ignore the noise coming from the randomness of the signals, that is, I assume $u_i^j = 0$ in (21). Moreover, I simplify the notation such that $\mathrm{E}[d_i]$ is the rational expectation from (18), and $\tilde{\mathrm{E}}^j[d_i]$ is the subjective dividend expectations from (19).[16]

With these assumptions, the subjective dividend expectation is:

$$\tilde{\mathrm{E}}^j[d_i] = \mathrm{E}[d_i] + c\kappa^j \omega_i. \tag{23}$$

[16]To be clear, $\mathrm{E}[d_i]$ is not the same as θ, but rather the rational expectation conditional on the two public signals. For example, if $\mu_0 = 1, \delta = \frac{1}{2}$, and $\theta_i = s_i^{max} = s_i^{min} = 2$, then the rational expectation from (18) is $\mathrm{E}[d_i] = \mathrm{E}[\theta_i | s_i^{max}, s_i^{min}] = 1.5$, which is below the true value of $\theta_i = 2$. The true value, however, is not observable, so the best an investor can do is to form the rational expectation.

Importantly, the optimism parameter κ^j varies across investors to allow for the realistic case where some market participants are too optimistic, others are too pessimistic, and others again are approximately unbiased.

Based on their subjective dividend expectations, each investor chooses a vector of portfolio weights, $\pi_j = [\pi_{j,1}, \pi_{j,2}, \dots \pi_{j,n}]'$, to maximize their mean-variance utility:

$$\max_{\pi_j} \pi_j' \left(\tilde{\mathrm{E}}^j[d] - p \right) - \frac{\gamma^j}{2} \pi_j' \Omega \pi_j, \tag{24}$$

where $\tilde{\mathrm{E}}^j[d]$ is the vector of subjective dividend expectations and p is the vector of prices.

The solution to the portfolio choice problem in (24) is:

$$\pi_j = \frac{1}{\gamma^j} \Omega^{-1} \left(\tilde{\mathrm{E}}[d] - p \right) \tag{25}$$

$$= \frac{1}{\gamma^j} \Omega^{-1} \left(\mathrm{E}[d] + c\kappa^j \omega - p \right), \tag{26}$$

where the second line expands $\tilde{\mathrm{E}}^j[d]$ using the expression in (23) and ω is the vector of ω_i's. Relative to a rational investor, an optimistic investor has excess demand for stocks with a high dividend volatility.

In equilibrium, supply equals demand, $\pi^* = \sum_j \pi_j$, so the demand follows by summing over the positions of all individual investors:

$$\pi^* = \sum_{j=1}^{J} \frac{1}{\gamma_j} \Omega^{-1} \left(\mathrm{E}[d] + c\, \kappa^j \omega - p \right) \tag{27}$$

$$= \frac{1}{\gamma} \Omega^{-1} \left(\mathrm{E}[d] + c\, \kappa\, \omega - p \right), \tag{28}$$

where $1/\gamma = \sum_j \frac{1}{\gamma^j}$ and $\kappa = \sum_j \frac{\gamma}{\gamma^j} \kappa^j$ is the risk-aversion weighted optimism. We can think of γ and κ as capturing, respectively, the risk aversion and optimism of the representative investor. In particular, the representative investor is optimistic if κ is positive.

The sign of κ depends, roughly, on the wealth controlled by optimists relative to pessimists.[17] The wealth controlled by rational investors does not affect the sign of κ but does affect its magnitude (the more wealth controlled by rational investors, the lower the effect of bias). For simplicity, think of κ as being positive if optimists control more wealth than pessimists. The evidence from Sharot (2011) suggests that optimists outnumber pessimists by four-to-one, so if the average wealth per person is the same in both groups, the total wealth of optimists is four times higher than that of pessimists.[18]

Equilibrium prices follow by solving for the market clearing price vector:

$$p = \mathrm{E}[d] + c\,\kappa\,(\omega - \bar{\omega}1) - \gamma\Omega\pi^*. \tag{29}$$

If the representative investor is rational, prices are correct for all stocks. If the representative investor is optimistic, stable stocks are undervalued, and volatile stocks are overvalued.

The subjective expected returns of the representative investor follow the standard CAPM:

$$\tilde{\mathrm{E}}[r_i] = \tilde{\mathrm{E}}[r_m]\beta_i, \tag{30}$$

where $\tilde{\mathrm{E}}$ denotes the expectation of the representative investor, r_m is the return of the market portfolio, and $\beta_i = \mathrm{cov}(r_i, r_m)/\mathrm{var}(r_m)$ is the stock's market beta. The relation in (30) implies that the representative investor requires a higher return for investing in stocks with a higher market beta—a well-known result when individual investors have mean-variance preferences (e.g., Frazzini and Pedersen (2014)). From the perspective of the representative investor, therefore, the subjective risk of a stock is captured by its market beta. The next proposition

[17] The statement that the sign of κ depends on the wealth controlled by optimists relative to pessimists is strictly true if all investors have the same relative risk aversion and if the severity (the absolute magnitude of κ^j) of optimism and pessimism is the same. A counter-example could be if pessimists control more wealth than optimists, but optimists are very optimistic, whereas pessimists are only slightly pessimistic.

[18] There is even reason to suspect that optimists on average control more wealth person than pessimists due to a "triumph of the optimists," whereby optimists are more likely to invest in *any* risky assets, which leads them to endogenously controlling more wealth (Dimson et al., 2002).

shows that objective expected returns differ from subjective expected returns:

Proposition 2 (The Optimism-Adjusted CAPM). *Objective expected returns follow the Optimism-Adjusted CAPM:*

$$\mathrm{E}[r_i] = \tilde{\mathrm{E}}[r_m]\beta_i - c\,\kappa\,\sigma_i, \tag{31}$$

where σ_i is the stock's return volatility.

Proposition 2 shows that objective expected returns increase in a stock's market beta because of the risk compensation required by the representative investor but decrease in a stock's return volatility if the representative investor has an optimism bias.

To understand the relationship between beta and objective expected returns, consider a cross-sectional regression of objective expected returns on beta:

$$\mathrm{E}[r_i] = \lambda_0 + \lambda_1\beta_i + \epsilon_i. \tag{32}$$

The slope coefficient is:

$$\lambda_1 = \tilde{\mathrm{E}}[r_m]\beta_i - c\,\kappa\psi_\sigma, \tag{33}$$

where $\psi_\sigma = \frac{\overline{\mathrm{cov}}(\sigma_i,\beta_i)}{\overline{\mathrm{var}}(\beta_i)}$ is the slope parameter from a regression of return volatility on market beta. Empirically, ψ_σ is strongly positive at 0.21, with a t-statistic of 48 (see Appendix A.4.2). In other words, high beta stocks tend to have high return volatility.

If the representative investor has an optimism bias, the model implies that the relationship between objective expected returns and market beta is weaker than the representative investor expects. The relationship can even be negative if the effect of optimism bias is sufficiently strong. As such, the model can explain why the CAPM fails to explain average realized returns even if it perfectly explains subjective risk and subjective expected returns.

5 Conclusion: mispricing disconnects risk from returns

Newer empirical asset pricing models are much better at explaining realized returns than traditional models like the CAPM. Nevertheless, I show that the traditional models are much better at explaining the subjective risk and return expectations of equity analysts. I provide evidence that this disconnect arises because traditional models align with return-subtracting mispricing while newer models, especially one based on machine learning, align with return-enhancing mispricing. In other words, investors have overoptimistic cash flow expectations for stocks with high CAPM risk, which lowers their realized returns, while the opposite is true for stocks with high ML "risk."

These results suggest that the CAPM is a good model of risk but fails to explain realized returns because risk is correlated with mispricing. I show that the negative correlation between risk and mispricing can be explained by a model based on a standard CAPM setup, except that some investors have an optimism bias. High-beta stocks leave more room for optimism-induced mispricing, which leads to a disconnect between risk and returns.

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A Appendix

The appendix contains proofs (Section A.1), details on the data construction (Section A.2), details on the construction of model risk estimates (Section A.3), results showing that cash flow uncertainty is increasing in market beta (Section A.4), the risk and mispricing coordinates of equity factors (Section A.5), and the characteristics that drive subjective risk (Section A.6).

A.1 Proofs

A.1.1 Proof of proposition 1

Proof of (22). I start by deriving the rational posterior expectation from (18). The three random variables of interest are the expected dividend $\theta_i \sim N(\mu_0, \tau_0^2\omega_i^2)$ and the two public signals $s_i^k|\theta_i \sim N(\theta_i, \tau_1^2\omega_i^2)$ with $k \in \{1, 2\}$. These three variables have a multivariate normal distribution if every linear combination $Y = a\theta_i + bs_1^i + cs_2^i$ of its components is normally distributed. To see that this is the case, I write each signal as the sum of theta and a random variable u_i^k: $Y = a\theta_i + b(\theta_i + u_i^1) + c(\theta_i + u_i^2) = \theta_i(a+b+c) + bu_i^1 + cu_i^2$. Hence, Y is a linear combination of three independent normally distributed variables, which is itself normally distributed. The prior distribution of the expected cash flows and the two public signals are, therefore, multivariate normal:

$$\begin{pmatrix} \theta_i \\ s_i^1 \\ s_i^2 \end{pmatrix} = N\left(\begin{bmatrix} \mu_0 \\ \mu_0 \\ \mu_0 \end{bmatrix}, \quad \omega_i^2 \begin{bmatrix} \tau_0^2 & \tau_0^2 & \tau_0^2 \\ \tau_0^2 & \tau_0^2 + \tau_1^2 & \tau_0^2 \\ \tau_0^2 & \tau_0^2 & \tau_0^2 + \tau_1^2 \end{bmatrix}\right) \tag{A.1}$$

From the properties of a multivariate normal distribution, the conditional distribution is:

$$
\begin{aligned}
E[\theta_i|s_i^1, s_i^2] &= \mu_0 + \frac{\omega_i^2}{\omega_i^2} \begin{bmatrix} \tau_0^2 & \tau_0^2 \end{bmatrix} \begin{bmatrix} \tau_0^2 + \tau_1^2 & \tau_0^2 \\ \tau_0^2 & \tau_0^2 + \tau_1^2 \end{bmatrix}^{-1} \begin{bmatrix} s_i^1 - \mu_0 \\ s_i^2 - \mu_0 \end{bmatrix} \\
&= \mu_0 + \frac{1}{(\tau_0^2 + \tau_1^2)(\tau_0^2 + \tau_1^2) - \tau_0^4} \begin{bmatrix} \tau_0^2 & \tau_0^2 \end{bmatrix} \begin{bmatrix} \tau_0^2 + \tau_1^2 & -\tau_0^2 \\ -\tau_0^2 & \tau_0^2 + \tau_1^2 \end{bmatrix} \begin{bmatrix} s_i^1 - \mu_0 \\ s_i^2 - \mu_0 \end{bmatrix} \\
&= \mu_0 + \frac{1}{(\tau_0^2 + \tau_1^2)(\tau_0^2 + \tau_1^2) - \tau_0^4} \begin{bmatrix} \tau_0^2(\tau_0^2 + \tau_1^2 - \tau_0^2) & \tau_0^2(\tau_0^2 + \tau_1^2 - \tau_0^2) \end{bmatrix} \begin{bmatrix} s_i^1 - \mu_0 \\ s_i^2 - \mu_0 \end{bmatrix} \\
&= \mu_0 + \frac{1}{2}\frac{\tau_0^2}{\tau_0^2 + \tau_1^2/2}(s_i^1 - \mu_0) + \frac{1}{2}\frac{\tau_0^2}{\tau_0^2 + \tau_1^2/2}(s_i^2 - \mu_0) \\
&= \mu_0 + \delta\left(\frac{1}{2}s_i^{max} + \frac{1}{2}s_i^{min} - \mu_0\right) \\
&= (1-\delta)\mu_0 + \frac{\delta}{2}s_i^{max} + \frac{\delta}{2}s_i^{min},
\end{aligned}
$$

where $\delta = \frac{\tau_0^2}{\tau_0^2 + \tau_1^2/2}$ is a shrinkage parameter which is the same for all stocks, $s_i^{max} := \max(s_i^1, s_i^2)$ is the highest signal, and $s_i^{min} := \min(s_i^1, s_i^2)$ is the lowest signal. Hence, the rational posterior puts equal weight on the two signals.

Next, I show the expected bias when the investor's posterior expectation follows (19). I define the bias in investor inference as $b_j^i = \tilde{\mathrm{E}}^j[\theta_i|s_i^{max}, s_i^{min}, \kappa^j] - \mathrm{E}[\theta_i|s_i^{max}, s_i^{min}]$. This bias depends on the two random signals, but I want to characterize its expected value. To do so, I use the results from Nadarajah and Kotz (2008) about the expected value of the maximum and minimum of two normal random variables. If X_1 and X_2 are jointly normal with expected value μ_X and μ_Y, variances σ_X^2 and σ_Y^2, and covariance $\mathrm{cov}(X, Y)$, then their expected values are:

$$
\begin{aligned}
E[\max(X_1, X_2)] &= \mu_X \Phi\left(\frac{\mu_X - \mu_Y}{\xi}\right) + \mu_Y \Phi\left(\frac{\mu_Y - \mu_X}{\xi}\right) + \xi\phi\left(\frac{\mu_X - \mu_Y}{\xi}\right), \\
E[\min(X_1, X_2)] &= \mu_X \Phi\left(\frac{\mu_X - \mu_Y}{\xi}\right) + \mu_Y \Phi\left(\frac{\mu_Y - \mu_X}{\xi}\right) - \xi\phi\left(\frac{\mu_X - \mu_Y}{\xi}\right),
\end{aligned}
$$

where $\Phi(x)$ is the cumulative distribution function of a standard normal variable, $\phi(x)$ is the density function of a standard normal variable, and $\xi = \sqrt{\sigma_X^2 + \sigma_Y^2 - 2\text{cov}(X, Y)}$. If the two variables have the same expected mean μ and variance, σ^2, then we have:

$$E[\max(X_1, X_2)] = \mu + \xi\phi(0) \tag{A.2}$$

$$E[\min(X_1, X_2)] = \mu - \xi\phi(0) \tag{A.3}$$

Using this result, I can derive the expected bias from Proposition 1:

$$\begin{aligned}
\text{E}[b_i^j] &= \text{E}\left[(1-\delta)\mu_0 + \left(\frac{\delta}{2} + \kappa^j\right) s_i^{max} + \left(\frac{\delta}{2} - \kappa^j\right) s_i^{min} - \left((1-\delta)\mu_0 + \frac{\delta}{2} s_i^{max} + \frac{\delta}{2} s_i^{min}\right)\right] \\
&= \kappa^j(\text{E}[s_i^{max}] - \text{E}[s_i^{min}]) \\
&= \kappa^j 2\phi(0)\xi \\
&= \kappa^j 2\phi(0)\sqrt{2\omega_i^2(\tau_0^2 + \tau_1^2) - 2\omega_i^2\tau_0^2} \\
&= \kappa^j 2\phi(0)\sqrt{2}\tau_1\omega_i \\
&= c\kappa^j\omega_i
\end{aligned}$$

where $c = 2^{1.5}\phi(0)\tau_1 = 1.13\tau_1 > 0$ is a constant. On the fourth line, I evaluate $E[s_i^{max}]$ and $E[s_i^{min}]$ using (A.2) and (A.3), respectively; on the fifth line, I use the variance and covariance from (A.1) to expand ψ; and on the final line I define c. Furthermore, c is positive because I assume that τ_1 is positive, and the remaining components are positive constants. □

A.1.2 Proof of Proposition 2

Proof of (30). I first show how to recover the subjective expected excess return from the perspective of the representative investor from (30). Based on the vector of prices from all stocks in (29), the price of a single stock is:

$$\begin{aligned}
p_i &= e_i' p \\
&= e_i'(\mathrm{E}[d] + c\,\kappa\omega - \gamma\Omega\pi^*) \\
&= \mathrm{E}[d_i] + c\,\kappa\omega_i - \gamma e_i'\Omega\pi^* \\
&= \tilde{\mathrm{E}}[d_i] - \gamma e_i'\Omega\pi^*
\end{aligned}$$

where e_i is a vector where the ith entry is one and the remaining entries are zero, and the last line uses the definition of the subjective dividend expectation from (23), but from the perspective of the representative investor, whose expectations I denote by $\tilde{\mathrm{E}}$.

From the price of a single stock, we can infer its subjective expected excess return:

$$\begin{aligned}
\tilde{\mathrm{E}}[r_i] &= \frac{\tilde{\mathrm{E}}[d_i]}{p_i} - 1 \\
&= \gamma\frac{1}{p_i} e_i'\Omega\pi^* \\
&= \gamma\frac{1}{p_i}\mathrm{cov}(d_i, d'\pi^*) \\
&= \gamma\mathrm{cov}(r_i, d'\pi^*) \\
&= \gamma\mathrm{cov}(r_i, r_m)p'\pi^*,
\end{aligned}$$

where the first line defines the subjective expected excess return of stock i; the second line expands this expression using the price of a single stock; the third line uses that $e_i'\Omega\pi^*$ is equal to the divided covariance between a portfolio that invests one unit in stock i and a portfolio that invests in all stocks using weight of π^*; the fourth line moves $1/p_i$ inside the covariance, and uses that $d_i/p_i = 1 + r_i$ and that $\mathrm{cov}(1 + r_i, x) = \mathrm{cov}(r_i, x)$; the fifth line multiply and divide by the total market capitalization, $p'\pi^*$, moves $1/(p'\pi^*)$ inside the covariance, uses that $(d'\pi^*)/(p'\pi^*) = 1 + r_m$, and that $\mathrm{cov}(x, 1 + r_m) = \mathrm{cov}(x, r_m)$. The subjective expected excess return relationship holds for all assets, so it must hold for the market portfolio:

$$\tilde{\mathrm{E}}[r_m] = \gamma \sigma_m^2 p' \pi^*,$$

where σ_m^2 is the variance of the market portfolio. Re-arranging this expression we get:

$$\gamma p' \pi^* = \frac{\tilde{\mathrm{E}}[r_m]}{\sigma_m^2},$$

which we can insert into the subjective expected excess return for stock i:

$$\begin{aligned} \tilde{\mathrm{E}}[r_i] &= \tilde{\mathrm{E}}[r_m] \frac{\mathrm{cov}(r_i, r_m)}{\sigma_m^2} \\ &= \tilde{\mathrm{E}}[r_m] \beta_i, \end{aligned} \tag{A.4}$$

where the second line defines β_i. The equation in (A.4) coincides with (30).

□

Proof of (31). Next, I show how to recover the objective expected return from (31). The objective expected return of a stock is:

$$\begin{aligned} \tilde{\mathrm{E}}[r_i] &= \frac{\mathrm{E}[d_i]}{p_i} - 1 \\ &= \frac{\tilde{\mathrm{E}}[d_i] - c\,\kappa\omega_i}{p_i} - 1 \\ &= \tilde{\mathrm{E}}[r_i] - c\,\kappa \frac{\omega_i}{p_i} \\ &= \tilde{\mathrm{E}}[r_m]\beta_i - c\,\kappa \frac{\omega_i}{p_i} \\ &= \tilde{\mathrm{E}}[r_m]\beta_i - c\,\kappa\sigma_i. \end{aligned} \tag{A.5}$$

The first line defines the objective expected excess return; the second line expresses the objective expected dividend as the subjective expected dividend minus $c\,\kappa(\omega_i - \bar{\omega})$, which follows from (23); the third line uses the fact that $\tilde{\mathrm{E}}[d_i]/p_i - 1 = \tilde{\mathrm{E}}[r_i]$; the fourth line uses the subjective expected return relationship from (30); and the fifth line uses the fact that ω_i/p_i is equal to the stock's return volatility. □

A.2 Data

A.2.1 Realized stock returns and firm characteristics

I get realized stock returns and firm characteristics from the data set in Jensen et al. (2023), which I download from WRDS at wrds-www.wharton.upenn.edu/login/?next=/pages/get-data/contributed-data-forms/global-factor-data. I use realized returns from CRSP and accounting data from Compustat. I restrict the sample to ordinary common stocks listed on NYSE, NASDAQ, or AMEX, and I only retain the primary security of a firm. Specifically (using the column names in the Jensen et al. (2023) data set), I set `excntry`=1, `obs_main`=1, `common`=1, `exch_main`=1, `primary_sec`=1 and `source_crsp`=1.

A.2.2 Subjective expected returns

This section describes how I compute the subjective expected return from Value Line, Morningstar, and I/B/E/S. For all three providers, I adjust per share items for stock splits and other corporate actions using the `cfacshr` variable from CRSP. In addition, when converting a subjective return forecast to a different horizon, I use geometric compounding. For example, to convert the four-year expected return from Value Line to a one-year expected return, I use the formula:

$$\tilde{\mathrm{E}}_t[r_{t+1}] = (1 + \tilde{\mathrm{E}}_t[r_{t,t+4}])^{1/4} - 1, \tag{A.6}$$

where $\tilde{\mathrm{E}}_t[r_{t+1}]$ is the inferred one-year expected return and $\tilde{\mathrm{E}}_t[r_{t,t+4}]$ is the actual four-year expected return.

Value Line

Value Line provides a high and low price target for each stock in their coverage universe. I use the simple average of these two numbers as Value Line's price target. Moreover, Value Line provides long-term forecasts (including the price target) over a horizon of three-to-five years. Throughout the paper, I assume that three-to-five-year forecasts are four-year forecasts. To compute the subjective expected return from Value Line, I use their one-year dividend expectation, four-year dividend expectation, four-year price target, and the current price from CRSP. The specific procedure I use depends on whether the stock has non-zero dividend expectations.

For firms where the one- and four-year expected dividend is zero, the expected return is:

$$\tilde{\mathrm{E}}_t[r_{t,t+4}] = \frac{\tilde{\mathrm{E}}_t[p_{t+4}]}{p_t} - 1,$$

where $\tilde{\mathrm{E}}_t[r_{t,t+4}]$ is the expected return over the next four years, $\tilde{\mathrm{E}}_t[p^i_{t+4}]$ is the four-year price target, and p_t is the current price.

For stocks with non-zero dividends, I also need to account for the value of the dividends. Following Brav et al. (2005), I assume that the dividend is reinvested and thus grows at the expected returns. For example, for a stock with a subjective expected return of 10%, a dividend of 10 in year $t+3$ grows to $10(1+0.1) = 11$ in $t+4$.

For firms where the one-year expected dividend is zero, but the four-year expected dividend is non-zero, I assume that the expected dividend grows linearly over time, $\tilde{\mathrm{E}}_t[d_{t+1+k}] = \frac{k}{3}\tilde{\mathrm{E}}_t[d_{t+4}]$, where $\tilde{\mathrm{E}}_t[d_{t+h}]$ is the expected dividend h periods ahead. The annualized subjective return expectation is the value of $\tilde{\mathrm{E}}_t[r^*_{t+1}]$ that solves:

$$(1 + \tilde{\mathrm{E}}_t[r^*_{t+1}])^4 = \frac{\tilde{\mathrm{E}}_t[p_{t+4}]}{p_t} + \frac{\tilde{\mathrm{E}}_t[d_{t+2}](1 + \tilde{\mathrm{E}}_t[r^*_{t+1}])^2 + \tilde{\mathrm{E}}_t[d_{t+3}](1 + \tilde{\mathrm{E}}_t[r^*_{t+1}]) + \tilde{\mathrm{E}}_t[d_{t+3}]}{p_t},$$

and the four-year subjective expected return is then $\tilde{\mathrm{E}}_t[r_{t,t+4}] = (1 + \tilde{\mathrm{E}}_t[r^*_{t+1}])^4 - 1$.

For firms where both dividend expectations are non-zero, the subjective return expectation is the value of $\tilde{\mathrm{E}}_t[r^*_{t+1}]$ that solves:

$$(1+\tilde{\mathrm{E}}_t[r^*_{t+1}])^4 = \frac{\tilde{\mathrm{E}}_t[p_{t+4}]}{p_t} + \frac{\tilde{\mathrm{E}}_t[d_{t+1}]}{p_t}\frac{(1+\tilde{\mathrm{E}}_t[r^i_{t+1}])^4 - (1+\tilde{\mathrm{E}}_t[g_{t+4}])^4}{\tilde{\mathrm{E}}_t[r^*_{t+1}] - \tilde{\mathrm{E}}_t[g_{t+4}]}$$

where $\tilde{\mathrm{E}}_t[g_{t+4}]$ is the expected dividend growth from year t+1 to t+4 computed as:

$$\tilde{\mathrm{E}}_t[g_{t+4}] = \left(\frac{\tilde{\mathrm{E}}_t[d_{t+4}]}{\tilde{\mathrm{E}}_t[d_{t+1}]}\right)^{1/3} - 1,$$

and, again, the four-year subjective expected return is then $\tilde{\mathrm{E}}_t[r_{t,t+4}] = (1+\tilde{\mathrm{E}}_t[r^*_{t+1}])^4 - 1$.

Remark

Value Line provides their expected annualized total return in their reports. In figure A.1, it can be seen in the top left corner. However, in the data I received, there are errors in this data item before 2000. Specifically, when I look at old reports, the value for the expected return does not match those I have in my data. In contrast, the price target and dividend expectations match. After 2000, my implied return expectations match their data item almost perfectly. To ensure consistency, I compute the subjective expected return from the price target and dividend expectations throughout the sample.

I/B/E/S

The subjective expected return from I/B/E/S is:

$$\tilde{\mathrm{E}}_t[r_{t,t+1}] = \frac{\tilde{\mathrm{E}}_t[p_{t+1}] + \tilde{\mathrm{E}}_t[d_{t+1}]}{p_t}, \tag{A.7}$$

where $\tilde{\mathrm{E}}_t[p_{t+1}]$ and is the median consensus one-year price target from I/B/E/S, $\tilde{\mathrm{E}}_t[d^i_{t+1}]$ is the median consensus dividend forecast over the next fiscal year from I/B/E/S, and p^i_t is the stock's price at the day of the forecast from CRSP. If the dividend forecast from

I/B/E/S is unavailable, I use the one-year ahead dividend forecast from Value Line instead. If the Value Line forecast is also unavailable, I assume that the expected dividend is zero. Dividend expectations from I/B/E/S are only available for a broad cross-section of firms from 2002/05/16, so I use Value Line's dividend expectations before this date. The two series are highly similar, as the Spearman correlation between the implied dividend yield from I/B/E/S and the dividend yield from Value Line is 0.96.

Morningstar

Morningstar estimates a fair value for each stock in their coverage universe using a discounted cash flow methodology. The discount rate is the company's weighted average cost of capital, where the cost of equity described earlier is a key input together with the cost of debt. The fair value estimate represents the fair value today, so to get an estimate of forward-looking expected returns, I need an assumption about how quickly market prices will converge to the fair value. Morningstar generally expects the convergence to happen over three years, and I adopt this convention.[19]

Therefore, I create a three-year price target by compounding the fair value estimate using the cost of equity and dividing this price target by the current price:

$$\tilde{\mathrm{E}}_t[r_{t,t+3}] = \frac{(\text{fair value})_t \times (1 + \text{cost of equity}_t)^3}{p_t} - 1, \tag{A.8}$$

where p_t is the price from Morningstar. This convention means that the (annualized) subjective expected return is equal to the cost of equity if Morningstar perceives the stock to be fairly valued, above the cost of equity if Morningstar perceives the stock to be undervalued,

[19] From Morningstar's equity research methodology in 2022: "We expect that if our base-case assumptions are true the market price will converge on our fair value estimate over time, generally within three years (although it is impossible to predict the exact time frame in which market prices may adjust). If you bought a company's stock at exactly our fair value estimate today, we would expect that you should achieve total returns in line with our assumed cost of equity for the next three years, absent a change in business prospects relative to our base-case expectations. A stock price lower than our fair value estimate suggests that there is a higher probability" (Morningstar, 2022).

and below the cost of equity if Morningstar perceives the stock to be overvalued.

A.2.3 Earnings expectations from I/B/E/S and Value Line

I get subjective cash flow expectations from Value Line and I/B/E/S, primarily reflecting earnings per share (EPS) forecasts. From I/B/E/S, I extract the long-term growth in EPS forecasts (`fpi` of 0), annual EPS forecasts over the next two fiscal years (`fpi` of 1 and 2), and quarterly EPS forecasts over the next four fiscal quarters (`fpi` of 6, 7, 8 and 9). I always use the median forecast from the unadjusted consensus file.

From Value Line, I obtain annual EPS forecasts over the next two fiscal years and an EPS forecast over a three-to-five year horizon, which I assume reflects a forecast horizon of four years. In addition, I estimate Value Line's EPS forecast in fiscal year three by linear interpolation between the two- and four-year forecasts, and the EPS forecast in fiscal year five by linear extrapolation from the same two points.[20]

For each firm-fiscal year pair, I only retain the first EPS forecast issued at least 45 days and no more than 180 days after the announcement of the previous fiscal year. This gap ensures that the forecast has had time to reflect the previous fiscal year's information. I get earnings announcement dates from I/B/E/S. Finally, I compute forecast errors using the EPS realization from the I/B/E/S unadjusted "actuals" file, and I winsorize all forecast errors at the top/bottom 1% to limit the influence of outliers. I adjust all forecasted and realized EPS for stock splits and corporate actions using the `cfacshr` variable from CRSP.

A.2.4 Combining data sets

The identifier in the Value Line data is a stock's eight-digit CUSIP and ticker. I merge this data with CRSP using the `crsp.stocknames` table on WRDS. I match securities on historical CUSIPs; if this match fails, I use header CUSIPs and historical tickers. To ensure

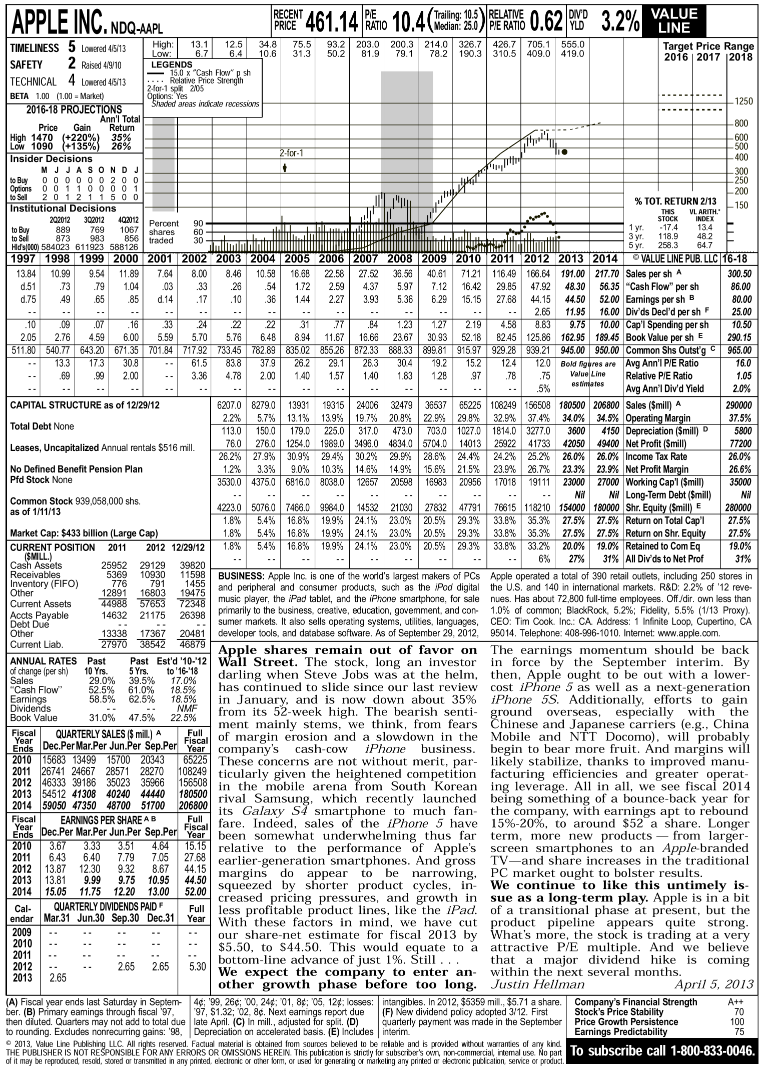
[20]Specifically, I estimate Value Line's three- and five-year EPS forecast as $\tilde{\mathrm{E}}_t[eps^i_{t+h}] = \tilde{\mathrm{E}}_t[eps^i_{t+2}] + (\tilde{\mathrm{E}}_t[eps^i_{t+4}] - \tilde{\mathrm{E}}_t[eps^i_{t+4}])(h-2)$ where $\tilde{\mathrm{E}}_t[eps^i_{t+h}]$ is Value Line's EPS forecast for fiscal year $t+h$.

that I have correctly merged Value Line and CRSP, I require that the stock price from Value Line is within 5% of the most recent stock price from CRSP. In total, I match 95% of the Value Line observations to CRSP.

The identifier I use from Morningstar Direct is the underlying firm's CIK code and the stock's ticker. I first merge the data to Compustat using the CIK values from the `comp.funda` table on WRDS. From Compustat, I get a historical six-digit CUSIP that identifies the firm. Then, I use the six-digit CUSIP from Compustat and the ticker from Morningstar to merge the data with CRSP using the `crsp.stocknames` table on WRDS. To ensure that I have correctly merged Morningstar and CRSP, I require that the stock price from Morningstar is within 5% of the most recent stock price from CRSP. In total, I match 65% of the Morningstar observations to CRSP.

Finally, I use the `wrdsapps.ibcrsphist` table from WRDS to link CRSP and I/B/E/S data, and the `crsp.ccmxpf_lnkhist` table from WRDS to link CRSP and Compustat.

A.2.5 Example of a Value Line investment report



APPLE INC. NDQ-AAPL

RECENT PRICE 461.14 | P/E RATIO 10.4 (Trailing: 10.5 Median: 25.0) | RELATIVE P/E RATIO 0.62 | DIV'D YLD 3.2% | VALUE LINE

TIMELINESS 5 Lowered 4/5/13
SAFETY 2 Raised 4/9/10
TECHNICAL 4 Lowered 4/5/13
BETA 1.00 (1.00 = Market)

High:	13.1	12.5	34.8	75.5	93.2	203.0	200.3	214.0	326.7	426.7	705.1	555.0
Low:	6.7	6.4	10.6	31.3	50.2	81.9	79.1	78.2	190.3	310.5	409.0	419.0

Target Price Range 2016 | 2017 | 2018

LEGENDS
15.0 x "Cash Flow" p sh
.... Relative Price Strength
2-for-1 split 2/05
Options: Yes
Shaded areas indicate recessions

2016-18 PROJECTIONS

	Price	Gain	Ann'l Total Return
High	1470	(+220%)	35%
Low	1090	(+135%)	26%

Insider Decisions

	M	J	J	A	S	O	N	D	J
to Buy	0	0	0	0	0	0	2	0	0
Options	0	0	1	1	0	0	0	0	1
to Sell	2	0	1	2	1	1	5	0	0

Institutional Decisions

	2Q2012	3Q2012	4Q2012
to Buy	889	769	1067
to Sell	873	983	856
Hld's(000)	584023	611923	588126

Percent shares traded 90 60 30

2-for-1

% TOT. RETURN 2/13

	THIS STOCK	VL ARITH.* INDEX
1 yr.	-17.4	13.4
3 yr.	118.9	48.2
5 yr.	258.3	64.7

1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	© VALUE LINE PUB. LLC	16-18
13.84	10.99	9.54	11.89	7.64	8.00	8.46	10.58	16.68	22.58	27.52	36.56	40.61	71.21	116.49	166.64	**191.00**	**217.70**	Sales per sh A	**300.50**
d.51	.73	.79	1.04	.03	.33	.26	.54	1.72	2.59	4.37	5.97	7.12	16.42	29.85	47.92	**48.30**	**56.35**	"Cash Flow" per sh	**86.00**
d.75	.49	.65	.85	d.14	.17	.10	.36	1.44	2.27	3.93	5.36	6.29	15.15	27.68	44.15	**44.50**	**52.00**	Earnings per sh B	**80.00**
--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	2.65	**11.95**	**16.00**	Div'ds Decl'd per sh F	**25.00**
.10	.09	.07	.16	.33	.24	.22	.22	.31	.77	.84	1.23	1.27	2.19	4.58	8.83	**9.75**	**10.00**	Cap'l Spending per sh	**10.50**
2.05	2.76	4.59	6.00	5.59	5.70	5.76	6.48	8.94	11.67	16.66	23.67	30.93	52.18	82.45	125.86	**162.95**	**189.45**	Book Value per sh E	**290.15**
511.80	540.77	643.20	671.35	701.84	717.92	733.45	782.89	835.02	855.26	872.33	888.33	899.81	915.97	929.28	939.21	**945.00**	**950.00**	Common Shs Outst'g C	**965.00**
--	13.3	17.3	30.8	--	61.5	83.8	37.9	26.2	29.1	26.3	30.4	19.2	15.2	12.4	12.0	Bold figures are		Avg Ann'l P/E Ratio	**16.0**
--	.69	.99	2.00	--	3.36	4.78	2.00	1.40	1.57	1.40	1.83	1.28	.97	.78	.75	Value Line		Relative P/E Ratio	**1.05**
--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	.5%	estimates		Avg Ann'l Div'd Yield	**2.0%**
						6207.0	8279.0	13931	19315	24006	32479	36537	65225	108249	156508	**180500**	**206800**	Sales ($mill) A	**290000**
						2.2%	5.7%	13.1%	13.9%	19.7%	20.8%	22.9%	29.8%	32.9%	37.4%	**34.0%**	**34.5%**	Operating Margin	**37.5%**
						113.0	150.0	179.0	225.0	317.0	473.0	703.0	1027.0	1814.0	3277.0	**3600**	**4150**	Depreciation ($mill) D	**5800**
						76.0	276.0	1254.0	1989.0	3496.0	4834.0	5704.0	14013	25922	41733	**42050**	**49400**	Net Profit ($mill)	**77200**
						26.2%	27.9%	30.9%	29.4%	30.2%	29.9%	28.6%	24.4%	24.2%	25.2%	**26.0%**	**26.0%**	Income Tax Rate	**26.0%**
						1.2%	3.3%	9.0%	10.3%	14.6%	14.9%	15.6%	21.5%	23.9%	26.7%	**23.3%**	**23.9%**	Net Profit Margin	**26.6%**
						3530.0	4375.0	6816.0	8038.0	12657	20598	16983	20956	17018	19111	**23000**	**27000**	Working Cap'l ($mill)	**35000**
						--	--	--	--	--	--	--	--	--	--	**Nil**	**Nil**	Long-Term Debt ($mill)	**Nil**
						4223.0	5076.0	7466.0	9984.0	14532	21030	27832	47791	76615	118210	**154000**	**180000**	Shr. Equity ($mill) E	**280000**
						1.8%	5.4%	16.8%	19.9%	24.1%	23.0%	20.5%	29.3%	33.8%	35.3%	**27.5%**	**27.5%**	Return on Total Cap'l	**27.5%**
						1.8%	5.4%	16.8%	19.9%	24.1%	23.0%	20.5%	29.3%	33.8%	35.3%	**27.5%**	**27.5%**	Return on Shr. Equity	**27.5%**
						1.8%	5.4%	16.8%	19.9%	24.1%	23.0%	20.5%	29.3%	33.8%	33.2%	**20.0%**	**19.0%**	Retained to Com Eq	**19.0%**
						--	--	--	--	--	--	--	--	--	6%	**27%**	**31%**	All Div'ds to Net Prof	**31%**

CAPITAL STRUCTURE as of 12/29/12
Total Debt None
Leases, Uncapitalized Annual rentals $516 mill.
No Defined Benefit Pension Plan
Pfd Stock None
Common Stock 939,058,000 shs. as of 1/11/13
Market Cap: $433 billion (Large Cap)

CURRENT POSITION ($MILL.)	2011	2012	12/29/12
Cash Assets	25952	29129	39820
Receivables	5369	10930	11598
Inventory (FIFO)	776	791	1455
Other	12891	16803	19475
Current Assets	44988	57653	72348
Accts Payable	14632	21175	26398
Debt Due	--	--	--
Other	13338	17367	20481
Current Liab.	27970	38542	46879

ANNUAL RATES of change (per sh)	Past 10 Yrs.	Past 5 Yrs.	Est'd '10-'12 to '16-'18
Sales	29.0%	39.5%	17.0%
"Cash Flow"	52.5%	61.0%	18.5%
Earnings	58.5%	62.5%	18.5%
Dividends	--	--	NMF
Book Value	31.0%	47.5%	22.5%

Fiscal Year Ends	QUARTERLY SALES ($ mill.) A Dec.Per	Mar.Per	Jun.Per	Sep.Per	Full Fiscal Year
2010	15683	13499	15700	20343	65225
2011	26741	24667	28571	28270	108249
2012	46333	39186	35023	35966	156508
2013	54512	**41308**	**40240**	**44440**	**180500**
2014	**59050**	**47350**	**48700**	**51700**	**206800**

Fiscal Year Ends	EARNINGS PER SHARE A B Dec.Per	Mar.Per	Jun.Per	Sep.Per	Full Fiscal Year
2010	3.67	3.33	3.51	4.64	15.15
2011	6.43	6.40	7.79	7.05	27.68
2012	13.87	12.30	9.32	8.67	44.15
2013	13.81	**9.99**	**9.75**	**10.95**	**44.50**
2014	**15.05**	**11.75**	**12.20**	**13.00**	**52.00**

Cal-endar	QUARTERLY DIVIDENDS PAID F Mar.31	Jun.30	Sep.30	Dec.31	Full Year
2009	--	--	--	--	--
2010	--	--	--	--	--
2011	--	--	--	--	--
2012	--	--	2.65	2.65	5.30
2013	2.65				

BUSINESS: Apple Inc. is one of the world's largest makers of PCs and peripheral and consumer products, such as the *iPod* digital music player, the *iPad* tablet, and the *iPhone* smartphone, for sale primarily to the business, creative, education, government, and consumer markets. It also sells operating systems, utilities, languages, developer tools, and database software. As of September 29, 2012, Apple operated a total of 390 retail outlets, including 250 stores in the U.S. and 140 in international markets. R&D: 2.2% of '12 revenues. Has about 72,800 full-time employees. Off./dir. own less than 1.0% of common; BlackRock, 5.2%; Fidelity, 5.5% (1/13 Proxy). CEO: Tim Cook. Inc.: CA. Address: 1 Infinite Loop, Cupertino, CA 95014. Telephone: 408-996-1010. Internet: www.apple.com.

Apple shares remain out of favor on Wall Street. The stock, long an investor darling when Steve Jobs was at the helm, has continued to slide since our last review in January, and is now down about 35% from its 52-week high. The bearish sentiment mainly stems, we think, from fears of margin erosion and a slowdown in the company's cash-cow *iPhone* business. These concerns are not without merit, particularly given the heightened competition in the mobile arena from South Korean rival Samsung, which recently launched its *Galaxy S4* smartphone to much fanfare. Indeed, sales of the *iPhone 5* have been somewhat underwhelming thus far relative to the performance of Apple's earlier-generation smartphones. And gross margins do appear to be narrowing, squeezed by shorter product cycles, increased pricing pressures, and growth in less profitable product lines, like the *iPad*. With these factors in mind, we have cut our share-net estimate for fiscal 2013 by $5.50, to $44.50. This would equate to a bottom-line advance of just 1%. Still . . .

We expect the company to enter another growth phase before too long. The earnings momentum should be back in force by the September interim. By then, Apple ought to be out with a lower-cost *iPhone 5* as well as a next-generation *iPhone 5S*. Additionally, efforts to gain ground overseas, especially with the Chinese and Japanese carriers (e.g., China Mobile and NTT Docomo), will probably begin to bear more fruit. And margins will likely stabilize, thanks to improved manufacturing efficiencies and greater operating leverage. All in all, we see fiscal 2014 being something of a bounce-back year for the company, with earnings apt to rebound 15%-20%, to around $52 a share. Longer term, more new products — from larger-screen smartphones to an *Apple*-branded TV—and share increases in the traditional PC market ought to bolster results.

We continue to like this untimely issue as a long-term play. Apple is in a bit of a transitional phase at present, but the product pipeline appears quite strong. What's more, the stock is trading at a very attractive P/E multiple. And we believe that a major dividend hike is coming within the next several months.

Justin Hellman *April 5, 2013*

(A) Fiscal year ends last Saturday in September. **(B)** Primary earnings through fiscal '97, then diluted. Quarters may not add to total due to rounding. Excludes nonrecurring gains: '98, 4¢; '99, 26¢; '00, 24¢; '01, 8¢; '05, 12¢; losses: '97, $1.32; '02, 8¢. Next earnings report due late April. **(C)** In mill., adjusted for split. **(D)** Depreciation on accelerated basis. **(E)** Includes intangibles. In 2012, $5359 mill., $5.71 a share. **(F)** New dividend policy adopted 3/12. First quarterly payment was made in the September interim.

Company's Financial Strength	A++
Stock's Price Stability	70
Price Growth Persistence	100
Earnings Predictability	75

Figure A.1: Value Line investment report on Apple

Note: The figure shows an example of a report from the Value Line Investment Survey for Apple. The example also displays most of the information from the data set used in this paper. For example, the key subjective risk measure is the safety rank in the top left corner. I transform the safety rank into a continuous measure by taking an average of its subcomponent, financial strength, and price stability, shown in the bottom left corner. The primary input for the subjective return expectation is the average of the price range projections visible just below the safety rank box.

A.3 Model risk

A.3.1 Build pricing factors

My approach to creating factor risk in (2), requires me to know the exact weights of all stocks for each specific pricing factor. For example, to create the conditional covariance between a stock and tangency portfolio implied by the CAPM, I need to know the exact weights of all stocks in the market factor. Because of this structure, I cannot simply download pricing factors returns from, say, Kenneth French's data library, but rather need to build the market factors from scratch. This section describes how I build the pricing factors for each of the six factor models (CAPM, FF3, FF5, HXZ, SY, DHS). My guiding principle is to stay as close as possible to the factor creation methodology from the paper reference listed in Table 1.

The pricing factors are based on a common data set. Specifically, I start from the data set in Jensen et al. (2023) and apply the screens described in Section A.2.1. I further restrict the sample to non-microcap stocks, i.e., those with a market cap above the 20th percentile of NYSE stocks. In the Jensen et al. (2023) data set, this screens amounts to requiring that `size_grp` ∈ {`mega, large, small`}. I impose the size screen because Value Line and Morningstar overwhelmingly cover non-microcap stocks, but the screen has a negligible effect in practice because all the pricing factors I consider use value-weights, which means that non-microcap stocks get a tiny weight.

Table A.1 gives an overview of the pricing factors and in which model they are used. The first model I consider is the CAPM of Sharpe (1964), Lintner (1965), and Mossin (1966). The CAPM has one factor, the market factor, which I create as the value-weighted excess returns of all stocks in the sample. The market factor is used in all the seven factor models I consider.

The second model is the 3-factor model from Fama and French (1993, FF3), which adds a value and size factor to the CAPM. The size and value factor is created by independently

Table A.1: Pricing factors

Factor name	Factor type	Used in	Type	Sorting characteristic
MKT	Market	CAPM, FF3, CH4, FF5 HXZ4, SY4, DHS3	Long	
SMB (FF3)	Size	FF3	LS	`market_equity`
HML	Value	FF3, CH4, FF5	LS	`be_me`
UMD	Momentum	CH4	LS	`ret_12_1`
SMB (CH4)	Size	CH4	LS	`market_equity`
RMW	Profitability	FF5	LS	`ope_be`
INV	Investment	FF5, HXZ4	LS	`at_gr1`
SMB (FF5)	Size	FF5	LS	`market_equity`
ROE	Profitability	HXZ4	LS	`roeq`*
SMB (HXZ4)	Size	HXZ4	LS	`market_equity`
MGMT	MP-management	SY4	LS	`mispricing_mgmt`
PERF	MP-performance	SY4	LS	`mispricing_perf`
SMB (SY4)	Size	SY4	LS	`market_equity`
FIN	Equity issuance	DHS3	LS	`dhs_issuance`*
PEAD	Post-earnings ann. drift	DHS3	LS	`ear`*

Note: The table shows the pricing factors used in the eight asset pricing models and information about how they are created. *Factor name* has the factor abbreviation, which mostly follows those in the paper that proposed the asset pricing model. *Factor type* shows the economic idea underlying the factor. *Used in* shows the asset pricing models that use a specific factor. *Type* shows whether the factor is long only or long/short (LS). *Sorting characteristics* shows the stock characteristics used to sort stocks into the long and short leg, using the name from the data set in Jensen et al. (2023). A subscript of * means that the characteristic is not (currently) in the Jensen et al. (2023) data set, so I describe its construction more thoroughly in the text. All factors are based on value-weights.

sorting stocks into two size groups and three book-to-market groups. The size breakpoint is the median market equity among NYSE stocks, and the book-to-market breakpoints are the 30^{th} and 70^{th} percentiles of book-to-market for NYSE stocks. The intersection of these groups produces six portfolios, and the excess return of each portfolio is computed using value-weights. The size factor is the average of the three small stock portfolios minus the average of the three big stock portfolios. The value factor is the average of the two high book-to-market portfolios minus the average of the two low book-to-market portfolios.

The third model is the four-factor model from Carhart (1997, CH4), which adds a momentum factor to the three factors from FF3. The value factor is the same as in FF3. The momentum factor is created in the same way as the value factor, that is, by a double sort with size, except that the second sorting variable is a stock's realized return over the past 12 months, skipping the most recent month. The CH4 size factor is the average of the three small stock portfolios from the value sort and the three small stock portfolios from the momentum sort minus the average of the three big stock portfolios from the value sort and the three big stock portfolios from the momentum sort.

The fourth model is the five-factor model from Fama and French (2015, FF5). This model is motivated by the dividend discount model and adds a profitability and investment factor to FF3. The profitability and investment factors are created in the same way as the value and momentum factors, except that the second sorting variable is a stock's operating profitability and the negative of a stock's past 1-year asset growth, respectively. The size factor is the average of the small stock portfolios from the value, momentum, profitability, and investment sort minus the average of the corresponding large stock portfolios.

The fifth model is the investment CAPM from Hou et al. (2015, HXZ4). This model is motivated by the q theory of investments and adds a size, profitability, and investment factor to the CAPM. The profitability factor is created as the non-size factors from the Fama-French models, except that the second sorting variable is a stock's quarterly return on equity. Following Hou et al. (2015), the quarterly return on equity is computed as the quarterly net income divided by the book equity from the previous quarter, which I assume is available to investors immediately after the earnings announcement day (`RDQ` in Compustat).[21] The investment factor is exactly the same as in FF5. The size factor is created as in FF5, except that I use the size portfolios from the HXZ4 investment and profitability sorts.

So far, the models I have considered have a rational justification for "why" they work.

[21] The data set in Jensen et al. (2023) has a quarterly return on equity variable `roeq`, but it is assumed to be available to investors four months after the fiscal year-end, which is typically two months after the earnings announcement date. The lag convention makes a sizable difference in the ability of HXZ4 to explain realized returns, so I chose to create a new variable that is constructed in the exact same way as in Hou et al. (2015)

By contrast, the two remaining models have a behavioral justification. The sixth model I consider is the four-factor model from Stambaugh and Yuan (2017, SY4), which is motivated by persistent mispricing, especially overvaluation, which rational investors fail to correct due to arbitrage asymmetry. The model uses a market factor, a size factor, and two mispricing factors related to managerial actions and firm performance. The managerial and performance mispricing factors are created similarly to the Fama-French factors, except that the second sorting variable is a composite of mispricing factors related to managerial actions and firm performance, respectively, and that the breakpoints for these variables are the 20$^{\text{th}}$ and 80$^{\text{th}}$ percentiles among NYSE stocks. The size factor is created as in FF5, except that I use the size portfolios from the SY4 managerial and performance sorts.

The seventh and final model is the three-factor model from Daniel et al. (2020, DHS3), designed to capture short- and long-horizon mispricing due to investor biases. The model uses a market factor and two behavioral factors. The first behavioral factor is based on the post-earnings announcement drift and captures short-run mispricing. The factor is created in the same way as the SY4 factors, except that the second sorting variable is a stock's average return in excess of the market on the day before, the day of, and the two days after its most recent earnings announcement date (as indicated by `RDQ` in Compustat). The second behavioral factor is a equity financing factor and captures long-run mispricing. The financing factor is based on the 1-year net share issuance (NSI) from Pontiff and Woodgate (2008), and the 5-year composite share issuance (CSI) measure of Daniel and Titman (2006). The sorting procedure is unique to this factor. First, stocks are sorted into three CSI groups using the 20$^{\text{th}}$ percentile among NYSE stocks. Second, stocks with a negative NSI (firms that repurchase shares) are sorted into two groups using the NYSE median breakpoint. Stocks with a positive NSI (firms that issue shares) are sorted into three groups using the NYSE 30$^{\text{th}}$ and 70$^{\text{th}}$ breakpoint. The "low" NSI group are stocks in the repurchasing group with the most negative NSI group, and the "high" NSI group are stocks in the issuance group with the highest NSI. Finally, stocks are assigned to one of three financing groups. High financing stocks are those in the high groups for both NSI and CSI, or to the high group of NSI if CSI is missing, or to the high group of CSI if NSI is missing. Low financing stocks are those in the low groups for both NSI and CSI or to the low group by one group while missing the other. I then create six portfolios based on the interaction between the financing groups and two size groups (above or below the NYSE median). The financing factor is the average of the two low-financing portfolios minus the average of the two high-financing portfolios.

A.3.2 Tangency portfolio of factor models

Figure A.2 shows the tangency portfolio weights for the factor-based asset pricing models. I compute these weights by computing the average returns of the model's pricing factors, μ, and their variance-covariance matrix, Σ, over the full sample from 1972 to 2021, and then the weights are

$$\pi = \frac{1}{1'\Sigma^{-1}\mu}\Sigma^{-1}\mu, \tag{A.9}$$

where 1 is a conformable vector of ones.

Figure A.3 shows that the realized return coefficient used to rank models in Section 2.1 gives a similar ranking as if the models were ranked by the Sharpe ratio of their Tangency portfolio (which is the more standard approach following Barillas and Shanken (2017)).

A.3.3 Machine learning risk via XGBoost

In this section, I describe the approach I use to make machine learning predictions for the ML model from Section 1.2.2. I use the XGBoost model from Chen and Guestrin (2016), with the 153 characteristics from Jensen et al. (2023) as the independent variables, and the dependent variable is a stock's realized excess return over the next month.

The first model is based on training data from 1952 to 1971. However, XGBoost requires me to decide on several "hyper-parameters" such as the number of decision trees to use in

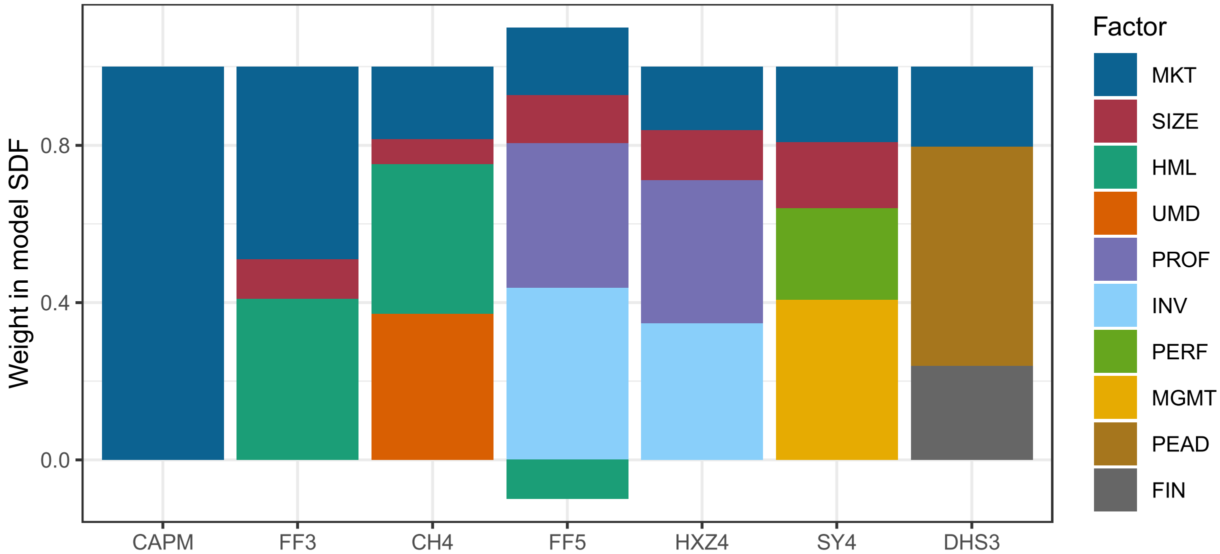


Figure A.2: Model-implied SDF weights

Note: The figures show the tangency portfolio weights for the factor-based asset pricing models. I compute these weights by computing the average returns of the model's pricing factors, μ, and their variance-covariance matrix, Σ, over the full sample from 1972 to 2021, and then the weights are

$$\pi = \frac{1}{1'\Sigma^{-1}\mu}\Sigma^{-1}\mu,$$

where 1 is a conformable vector of ones.

the ensemble and the maximum tree depth. To choose these hyper-parameters, I use the last ten years of the training period as the validation period. For a set of hyper-parameters, I train the model on the data prior to the validation period and record its mean squared error (MSE) on the validation data. I repeat this procedure for 20 sets of hyper-parameters shown in Table A.2 and choose the set of hyper-parameters with the lowest MSE. Using these hyper-parameters, I then re-train the model on the full training data, which gives me the first model, $\hat{f}_1(x_{i,t})$, where $x_{i,t} \in \mathbb{R}^{153\times 1}$ is the vector of stock characteristics. I then use $\hat{f}_1(x_{i,t})$ to predict returns from 1972-1981.

I update the model each decade by expanding the training, validation, and test period by ten years. For example, the second model is based on 1952-1981 as the training period,

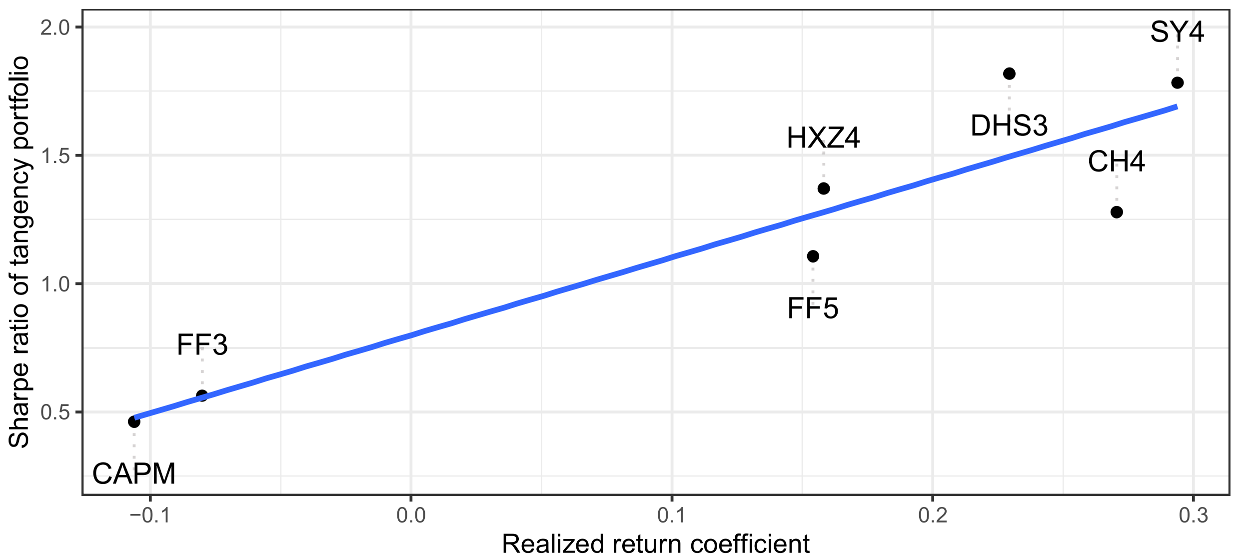


Figure A.3: Sharpe ratio versus realized return coefficient

Note: The figure shows the ability of the seven factor-based asset pricing models to explain realized returns inferred from different metric. The x-coordinate shows the realized return coefficient, that I also use in Figure 1. The y-coordinate shows the realized Sharpe ratio of the model's tangency portfolios, which is a more standard way to rank asset pricing models (Barillas and Shanken, 2017).

1972-1981 as the validation period, and 1982-1991 as the test period. I fit five different models that predict returns out-of-sample from 1972 to 2021.

A.4 Market beta and cash flow uncertainty

A.4.1 Market beta correlate with cash flow uncertainty

Proposition 1 shows that an investor with an optimism bias is relatively more optimistic about the cash flows for stocks with a high dividend variance. In this section, I show that market beta correlates with cash flow uncertainty.

To measure cash flow uncertainty, I use the dispersion in analysts' one-year EPS forecasts following the approach in Diether et al. (2002). Specifically, I use the cash flow forecasts

Table A.2: XGBoost Hyper-parameters

	No. Features	Tree depth	Learning rate	Sample size	Penalty
1	103	3	0.017	0.20	0.05
2	105	3	0.011	0.91	2.33
3	26	1	0.222	0.99	0.02
4	12	1	0.046	0.27	52.44
5	16	2	0.012	0.93	48.81
6	49	1	0.153	0.38	1.55
7	28	2	0.293	0.65	72.28
8	18	2	0.059	0.87	33.11
9	83	1	0.103	0.95	77.43
10	32	3	0.224	0.40	9.85
11	88	2	0.110	0.36	0.21
12	95	2	0.024	0.71	33.64
13	103	2	0.012	0.57	2.49
14	80	3	0.067	0.44	0.19
15	40	2	0.061	0.64	2.31
16	118	2	0.202	0.64	0.01
17	7	2	0.011	0.54	0.02
18	81	1	0.108	0.72	0.08
19	94	2	0.292	0.75	2.27
20	66	2	0.026	0.71	0.06

Note: The table shows the hyper-parameters considered for the XGBoost model described in Section 1.2.2. "No. Features" is the number of randomly chosen features considered for each decision tree, "Tree depth" is the maximum depth of each decision tree, "Learning rate" is the weight each new tree gets in the ensemble, "Sample size" is the fraction of the observations randomly chosen to train each decision tree on, and "Penalty" is an L2 (ridge) penalty. I get the hyper-parameter sets by specifying a tolerable range for each hyper-parameter and then use the `grid_max_entropy` function from the dials package (`https://dials.tidymodels.org/`) to get 20 sets that aim to cover the associated parameter space.

from I/B/E/S consensus file and estimate dispersion as:

$$\text{EpsUnc}_{i,t} = \frac{\text{SD}_t[\text{eps}^j_{i,t+1}]}{\text{abs}(\overline{\text{eps}}_{i,t+1})}, \tag{A.10}$$

where $\text{eps}^j_{i,t+1}$ is the forecast from analyst j about the earnings per share in the next fiscal year for stock i, the numerator computes the standard deviation of these forecasts, and the denominator computes the absolute mean of these forecasts. I require that at least two analysts contribute to the consensus forecasts (otherwise, the standard deviation cannot be computed) and that the mean EPS forecast is not exactly zero (to avoid dividing by zero).

To test whether market risk correlates with earnings uncertainty, I sort stocks into ten portfolios based on their market beta with monthly re-balancing. For each portfolio-month pair, I compute the average earnings uncertainty and average these scores over time.

Computing the standard error of this estimate is complicated by the fact that earnings uncertainty is persistent. To circumvent this issue, I use an adjustment from (Cochrane, 2005, p.223), which is valid if earnings predictability at the portfolio level has an AR(1) structure,

$$\text{SE}(\bar{x}_k) = \sqrt{\frac{\text{var}(x_k)}{T} \times \frac{1+\rho_k}{1-\rho_k}}, \tag{A.11}$$

where x_k is the earning predictability in portfolio k, T is the number of time-periods, and ρ_k is the monthly autocorrelation of x_k.

Figure A.4 shows that earnings uncertainty strongly increases in market beta, thus validating the assumption that high beta stocks have more uncertain cash flows than low beta stocks.

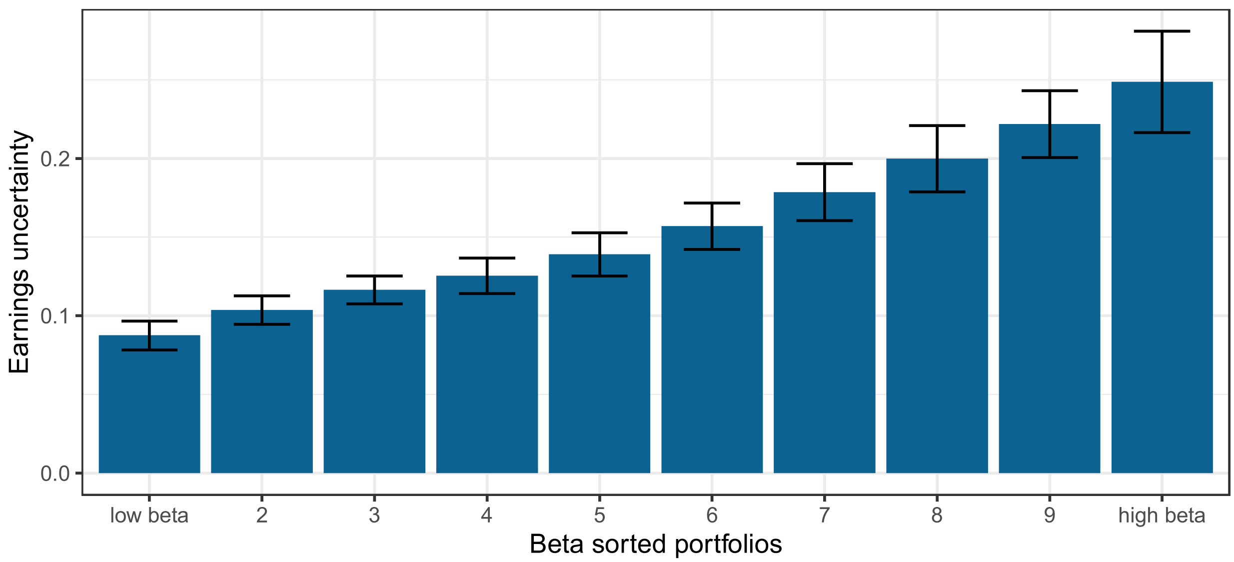


Figure A.4: Cash flow uncertainty increase in market beta

Note: The figure shows how earnings uncertainty relates to market beta. Earnings uncertainty is defined as the dispersion in analysts' one-year earnings forecasts, and stocks are sorted into ten portfolios based on their subjective risk. The error bars show 95% confidence intervals computed as $\pm 1.96 \times \mathrm{SE}(\bar{x}_k)$, where $\mathrm{SE}(\bar{x}_k)$ is defined in (A.11).

A.4.2 Market beta predict return volatility

The relationship between objective expected returns and market beta depends on the relationship between market beta and return volatility, as shown in Proposition 2. I estimate this relationship empirically by regressing total return volatility computed over the past 252 daily returns on the estimated market beta computed over the same period and time-fixed effects:

$$\sigma_{i,t} = \alpha_t + \psi_\sigma \beta_{i,t} + u_{i,t}, \tag{A.12}$$

where α_t is a month-fixed effect and ψ_σ is the empirical estimate of ψ_σ in (33).

Table A.3 column 1 shows that ψ_σ is positive and highly significant, with the estimate being 48 standard errors above zero. The evidence provides strong empirical support for the

Table A.3: Total return volatility increase in market beta and subjective risk

	Total return volatility
	(1)
$\hat{\beta}$	0.21
	(48.46)
Observations	1,152,879
R^2_{adj}	0.56
R^2_{adj} (within)	0.38

Note: The dependent variable is the annualized return volatility of a stock estimated from its past 252 daily returns. The explanatory variables are the stock market beta estimated from its past 252 daily returns ($\hat{\beta}$), and the regression includes year-month-fixed effects. The number in parentheses is the t-statistic of the estimated parameter based on standard errors clustered by firm and year-month.

assumption that return volatility increases in market beta.

A.5 Risk and mispricing coordinates of equity factors

Table A.4 shows risk and mispricing coordinates for the 119 equity factors from Jensen et al. (2023) used in Section 3.

Table A.4: Risk and mispricing coordinates of equity factors

Characteristic	Dir	Cluster	Risk-VL	Risk-MS	MP-FE	MP-EAR
age	-1	Low Leverage	25.9	12.3	-8.8	-9.6
ami_126d	1	Size	34.0	11.0	-16.0	0.6
at_gr1	-1	Investment	10.5	6.3	-6.3	3.8
be_gr1a	-1	Investment	4.1	6.4	-9.6	5.0
be_me	1	Value	12.0	8.9	-11.3	5.2
beta_60m	-1	Low Risk	-49.9	-25.1	6.3	8.8
betabab_1260d	-1	Low Risk	-40.1	-23.2	6.6	8.4
betadown_252d	-1	Low Risk	-34.0	-21.1	1.6	7.6

bev_mev	1	Value	13.0	9.7	-12.1	6.3
bidaskhl_21d	1	Low Leverage	68.4	24.3	-13.3	-12.5
capex_abn	-1	Debt Issuance	14.6	9.0	-3.0	-2.0
capx_gr2	-1	Investment	3.4	3.5	-0.7	5.0
capx_gr3	-1	Investment	2.3	3.9	-1.0	4.8
chcsho_12m	-1	Value	-29.6	-10.0	2.6	16.2
coa_gr1a	-1	Investment	6.2	2.8	-2.7	4.3
col_gr1a	-1	Investment	5.0	3.6	-6.6	-1.6
cop_atl1	1	Quality	-2.7	-8.2	8.6	8.8
corr_1260d	-1	Seasonality	16.2	1.2	-11.9	-2.1
coskew_21d	-1	Seasonality	3.0	0.7	0.4	-0.9
cowc_gr1a	-1	Accruals	2.9	-0.3	2.2	7.0
dbnetis_at	-1	Seasonality	19.6	6.5	5.9	1.0
debt_gr3	-1	Debt Issuance	10.1	4.9	5.2	0.7
debt_me	1	Value	6.5	6.3	-8.4	7.0
div12m_me	1	Value	-40.7	-13.4	7.7	6.1
dolvol_126d	-1	Size	22.3	8.2	-15.0	3.5
dolvol_var_126d	-1	Profitability	-30.1	-11.5	13.1	4.0
dsale_dinv	1	Profit Growth	4.1	1.2	5.8	-0.7
ebit_bev	1	Profitability	-35.0	-14.0	14.5	12.1
ebit_sale	1	Profitability	-38.8	-18.4	10.3	12.7
ebitda_mev	1	Value	-8.2	-0.9	1.3	13.1
emp_gr1	-1	Investment	-0.5	0.6	-4.8	4.6
eq_dur	-1	Value	-6.4	0.9	-2.5	10.3
eqnetis_at	-1	Value	-21.6	-9.4	0.2	15.6
eqnpo_12m	1	Value	-44.8	-13.4	6.5	15.2

eqnpo_me	1	Value	-40.9	-13.7	6.0	12.4
eqpo_me	1	Value	-41.5	-13.5	3.3	7.8
f_score	1	Profitability	-18.7	-11.1	12.2	6.9
fcf_me	1	Value	-16.0	-4.2	6.2	14.9
fnl_gr1a	-1	Debt Issuance	13.2	5.2	9.1	0.6
gp_at	1	Quality	1.8	-8.8	1.4	6.6
inv_gr1	-1	Investment	3.3	1.3	1.3	6.5
inv_gr1a	-1	Investment	4.6	4.9	2.0	3.2
iskew_ff3_21d	-1	Short-Term Reversal	-21.6	-8.9	-3.9	4.7
ivol_capm_252d	-1	Low Risk	-97.3	-24.9	13.0	13.3
ivol_ff3_21d	-1	Low Risk	-67.9	-22.0	13.3	14.1
kz_index	1	Seasonality	20.3	4.2	-11.1	2.7
lnoa_gr1a	-1	Investment	11.5	9.4	-1.9	2.8
lti_gr1a	-1	Seasonality	15.9	5.2	-5.8	-4.7
market_equity	-1	Size	47.2	18.3	-14.7	-3.1
mispricing_mgmt	1	Investment	-12.8	-2.2	2.4	10.5
mispricing_perf	1	Quality	-22.5	-17.4	12.4	10.9
ncoa_gr1a	-1	Investment	17.0	10.8	-3.7	1.9
netdebt_me	-1	Low Leverage	-2.5	0.3	7.2	-7.9
netis_at	-1	Value	-3.4	-6.3	5.7	13.9
nfna_gr1a	1	Debt Issuance	7.1	4.7	7.3	1.4
ni_be	1	Profitability	-43.6	-17.6	12.8	10.4
ni_me	1	Value	-36.8	-10.6	10.7	10.4
niq_at	1	Quality	-32.7	-19.7	12.1	11.4
niq_be	1	Profitability	-36.7	-17.0	13.2	11.4
niq_su	1	Profit Growth	-1.9	-0.2	11.9	1.1

nncoa_gr1a	-1	Investment	12.3	9.5	-2.7	4.0
noa_at	-1	Debt Issuance	-0.0	6.0	5.7	-1.5
noa_gr1a	-1	Investment	9.1	7.3	-0.6	7.2
o_score	-1	Profitability	-36.5	-17.4	11.3	8.4
oaccruals_at	-1	Accruals	16.3	4.6	-0.4	1.6
oaccruals_ni	-1	Accruals	17.0	5.2	-1.3	13.1
ocf_at	1	Profitability	-16.8	-14.3	9.8	15.0
ocf_at_chg1	1	Profit Growth	1.1	1.5	5.8	-1.3
ocf_me	1	Value	-3.6	-0.2	1.1	14.0
ocfq_saleq_std	-1	Low Risk	-16.8	-10.3	8.2	14.2
op_at	1	Quality	-9.8	-11.6	9.0	9.6
op_atl1	1	Quality	-9.5	-11.9	9.0	8.2
ope_be	1	Profitability	-22.6	-12.6	9.8	12.6
opex_at	1	Quality	8.9	1.2	-2.7	0.4
pi_nix	1	Seasonality	0.5	-0.8	5.7	4.7
ppeinv_gr1a	-1	Investment	3.6	4.2	-1.3	6.6
prc	-1	Size	72.1	19.9	-13.9	-9.8
prc_highprc_252d	1	Momentum	-46.3	-18.4	15.9	12.9
qmj	1	Quality	-32.7	-20.1	12.3	5.4
qmj_growth	1	Quality	-8.1	-2.5	8.4	-4.0
qmj_prof	1	Quality	-15.8	-15.7	11.4	12.2
qmj_safety	1	Quality	-46.3	-23.0	14.0	8.3
rd_me	1	Size	7.1	8.6	-4.5	-6.8
resff3_12_1	1	Momentum	-4.6	-0.1	12.2	3.4
resff3_6_1	1	Momentum	-4.7	-0.1	12.3	3.9
ret_12_1	1	Momentum	-9.6	-5.3	13.4	6.5

ret_12_7	1	Profit Growth	-7.2	-4.3	10.3	3.0
ret_1_0	-1	Short-Term Reversal	3.7	1.2	-13.5	-1.2
ret_3_1	1	Momentum	-3.9	-1.7	13.9	5.2
ret_60_12	-1	Investment	27.7	14.4	-11.4	-2.1
ret_6_1	1	Momentum	-6.1	-3.0	13.6	6.1
ret_9_1	1	Momentum	-8.0	-4.2	13.6	7.1
rmax1_21d	-1	Low Risk	-60.7	-23.5	11.0	13.1
rmax5_21d	-1	Low Risk	-65.7	-24.4	11.0	13.6
rmax5_rvol_21d	-1	Short-Term Reversal	8.0	2.7	-0.7	1.1
rskew_21d	-1	Short-Term Reversal	-17.9	-8.7	-0.9	6.4
rvol_21d	-1	Low Risk	-70.0	-24.8	12.5	14.7
sale_bev	1	Quality	5.2	1.1	0.8	4.3
sale_gr1	-1	Investment	0.3	0.6	-5.6	7.6
sale_gr3	-1	Investment	-2.3	-0.1	-1.9	5.8
sale_me	1	Value	13.8	6.4	-13.4	9.0
saleq_su	1	Profit Growth	0.1	-1.4	10.1	4.7
seas_11_15an	1	Seasonality	-1.3	-0.2	3.9	2.9
seas_16_20an	1	Seasonality	-1.3	-2.8	3.7	2.6
seas_16_20na	-1	Accruals	-3.1	1.2	-0.7	0.5
seas_1_1an	1	Profit Growth	-4.6	-2.8	9.0	1.9
seas_1_1na	1	Momentum	-0.0	-0.2	13.4	3.2
seas_2_5an	1	Seasonality	-2.9	-2.4	3.9	4.6
seas_2_5na	-1	Investment	0.0	1.3	-7.0	5.8
seas_6_10an	1	Seasonality	-2.4	-0.4	2.9	6.0
seas_6_10na	-1	Low Risk	-1.4	-2.7	-0.7	1.9
taccruals_at	-1	Accruals	9.4	1.0	-8.0	0.8

taccruals_ni	-1	Accruals	9.9	1.7	-8.4	9.1
tax_gr1a	1	Profit Growth	-8.1	-9.3	8.4	6.6
turnover_126d	-1	Low Risk	-28.3	-10.5	-2.8	13.1
turnover_var_126d	-1	Profitability	-28.3	-10.7	15.1	1.9
z_score	1	Low Leverage	-8.3	-10.2	12.8	3.0
zero_trades_126d	1	Low Risk	-26.8	-10.7	-6.1	13.4
zero_trades_252d	1	Low Risk	-26.6	-11.1	-6.2	13.6

Note: The table shows the relationship between the stock characteristics underlying the 119 equity factor from Jensen et al. (2023) and proxies for subjective risk and mispricing. *Characteristic* shows the name from the data set of Jensen et al. (2023). *Dir* shows the direction of the characteristic and realized returns (following the classification in the first paper that proposed the factor as in Jensen et al. (2023)). If the direction is negative is negative, the table shows the relationship between subjective risk/mispricing and the negative of the characteristic value. *Risk-VL*, *Risk-MS*, *MP-FE*, and *MP-EAR* shows, respectively, the t-statistic of the slope coefficient from a regression of subjective risk from Value Line, subjective risk from Morningstar, earnings announcement returns, and earnings forecast errors on the characteristic.

A.6 The drivers of subjective risk

To better understand the drivers of subjective risk, I estimate a model with eight stock characteristics as inputs, namely market beta, non-market volatility, Altman Z-score, market equity, book-to-market, past return from year $t-5$ to $t-1$, profitability, and asset growth:[22]:

$$\text{srisk}_{i,t} = \beta_0 + \sum_{k=1}^{8} \beta_k x_{i,t}^k + \epsilon_{i,t}, \tag{A.13}$$

[22]The characteristics are from the data set from Jensen et al. (2023), where they are called `beta_60m` (market beta), `ivol_capm_252d` (non-market volatility), `z_score` (Altman Z-score), `market_equity` (market equity), `be_me` (book-to-market), `ret_60_12` (past return), `ope_be` (profitability), and `at_gr1` (asset growth).

where Risk_t^i is either the subjective risk rating from Value Line or Morningstar, x^k denotes one of the eight characteristics. The characteristics are scaled to lie between -0.5 and 0.5, which means that the size of the parameter estimate is informative about the relative importance of the characteristic. Further, I compute standard errors clustered by firm and month to account for serial and cross-sectional correlation of the residuals.

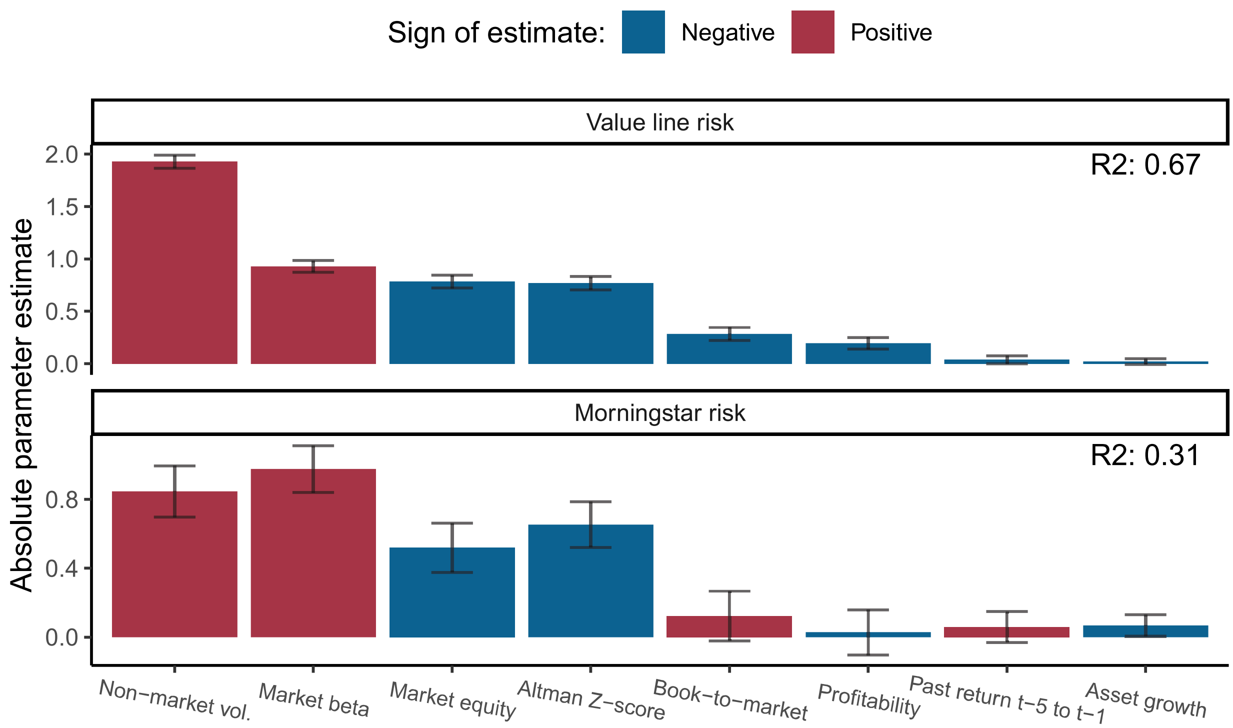


Figure A.5: Subjectively risky stocks are small, volatile, and distressed

Note: The figure shows the absolute parameter estimates from an OLS regression used to understand the drivers of subjective risk. The dependent variable in the upper and lower panel is, respectively, the safety rank from Value Line and the cost of equity from Morningstar, each standardized to have a cross-sectional mean of zero and variance of one. The explanatory variables are the eight characteristics from (A.13). The characteristics are scaled to lie between -0.5 and 0.5, so the absolute estimates are informative about their relative importance. The error bars show the 95% confidence interval of the estimates where the standard errors are clustered by firm and month. The characteristics are sorted based on the absolute value of the coefficient estimates from the Value Line model. The number in the top right corner shows the regression's adjusted R^2.

Figure A.5 shows the results. The upper panel shows the absolute coefficient estimate of (A.13) with the standardized safety rank from Value Line as the dependent variable. The results suggest that non-market volatility is the most important determinant of risk, followed by market beta, market equity, and the Altman Z-score. In contrast, book-to-market, profitability, past return, and asset growth are less important.

The lower panel of Figure A.5 shows the same regression with the cost of equity from Morningstar as the dependent variable. The results are qualitatively similar to those from Value Line, except that non-market volatility is less important and overtaken by market beta as the main determinant of risk. This finding could suggest that subjective risk from Morningstar reflects systematic risk to a larger extent than subjective risk from Value Line.

Non-market volatility is important for both subjective risk measures, which is somewhat surprising since it is commonly viewed as a measure of idiosyncratic risk that, according to asset pricing theory, should be less important than systematic risk. That said, the non-market volatility characteristic is the volatility of the residuals from a regression of a stock's returns on the market return. Besides truly stock-specific risk, the non-market volatility, therefore, also reflects a stock's exposure to non-market systematic risk factors.[23] This point is supported by the fact that Morningstar explicitly writes that their subjective risk measure only reflects systematic risk.

Overall, the results suggest that, according to Value Line and Morningstar, a subjectively risky stock is small (low market equity), volatile (high market beta and non-market volatility), and distressed (low Altman Z-score).

[23]As a side point, this argument is why I choose to call the characteristic "non-market volatility" even though it is often called "idiosyncratic volatility." Idiosyncratic volatility should arguably be reserved for return shocks that are independent across stocks. By contrast, the return shocks I use to compute the non-market volatility characteristics are uncorrelated with the market returns but almost surely correlated across stocks within the same industry, size groups, etc.